Problem Sheet 2
(LPP: Solution by Simplex Method)

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1. If $x_1 = 2,\ x_2 = 4$ and $x_3 = 1$ be a feasible solution to the system of equations

$$2x_1 - x_2 + 2x_3 = 2, \quad x_1 + 4x_2 = 18,$$

then find two basic feasible solutions.

Answer: $(x_1, x_2, x_3) = \left(\frac{26}{9}, \frac{34}{9}, 0\right), \quad \left(0, \frac{9}{2}, \frac{13}{4}\right)$.

2. The vector $(2, 1, 3)$ is a feasible solution of the set of equations

$$4x_1 + 2x_2 - 3x_3 = 1, \quad 6x_1 + 4x_2 - 5x_3 = 1.$$

Reduce the feasible solution to a basic feasible solution of the set.

Answer: $x_1 = 1, \quad x_2 = 0, \quad x_3 = 1.$

3. The vector $x_1 = 1,\ x_2 = 1,\ x_3 = 1,\ x_4 = 0$ is a feasible solution to the system of equations

$$x_1 + 2x_2 + 4x_3 + x_4 = 7, \quad 2x_1 - x_2 + 3x_3 - 2x_4 = 4.$$

Reduce the feasible solution to two different basic feasible solutions.

Answer: $(x_1, x_2, x_3, x_4) = \left(0, \frac{1}{2}, \frac{3}{2}, 0\right), \quad (3, 2, 0, 0).$

4. If $x_3 = 4$ and $x_4 = 8$ is the non-degenerate basic feasible solution to the linear programming problem:

$$\text{Maximize } Z = x_1 + 2x_2 \text{ subject to } \quad x_1 + 2x_2 + x_3 = 4; \quad x_1 + 4x_2 + x_4 = 8; \quad x_1, x_2, x_3, x_4 \geq 0,$$

then obtain the new basic feasible solution.

Answer: $x_1 = 4, \quad x_2 = 0, \quad x_3 = 0, \quad x_4 = 4.$

5. Show that $x_1 = 4,\ x_2 = 0,\ x_3 = 0,\ x_4 = 4$ is the optimal basic feasible solution to the linear programming problem

$$\text{Maximize } Z = x_1 + 2x_2 \text{ subject to } \quad x_1 + 2x_2 + x_3 = 4; \quad x_1 + 4x_2 + x_4 = 8; \quad x_1, x_2, x_3, x_4 \geq 0.$$

Answer: $x_1 = 4, \quad x_2 = 0, \quad x_3 = 0, \quad x_4 = 4.$

Solve the following problems using the Simplex Method.

1. (a) Maximize $Z = 3x_1 + 2x_2$ subject to $x_1 + x_2 \leq 4, \quad x_1 - x_2 \leq 2, \quad x_1, x_2 \geq 0.$
   Answer: $x_1 = 3,\ x_2 = 1, \quad \text{Max } Z = 11.$

   (b) Minimize $Z = x_1 - 3x_2 + 2x_3$ s.t. $3x_1 - x_2 + 2x_3 \leq 7, \ -2x_1 + 4x_2 \leq 12, \ -4x_1 + 3x_2 + 8x_3 \leq 10, \ x_1, x_2, x_3 \geq 0.$
   Answer: $x_1 = 4,\ x_2 = 5,\ x_3 = 0, \quad \text{Min } Z = -11$

   (c) Maximize $Z = 5x_1 + 10x_2 + 8x_3$ s.t. $3x_1 + 5x_2 + 2x_3 \leq 60, \quad 2x_1 + 4x_2 + 5x_3 \leq 100, \quad x_1, x_2, x_3 \geq 0.$
   Answer: $x_1 = 0,\ x_2 = 8,\ x_3 = 10, \quad \text{Max } Z = 160.$

   (d) Maximize $Z = 2x_1 + 4x_2 + x_3 + x_4$ s.t. $x_1 + 3x_2 + x_4 \leq 4, \ 2x_1 + x_2 \leq 3, \ x_2 + 4x_3 + x_4 \leq 3, \ x_1, x_2, x_3, x_4 \geq 0.$
   Answer: $x_1 = 1,\ x_2 = 1,\ x_3 = \frac{1}{2},\ x_4 = 0, \quad \text{Max } Z = \frac{13}{2}.$

2. (a) Maximize $Z = 7x_1 + 5x_2$ s.t. $-x_1 - 2x_2 \geq -6, \quad 4x_1 + 3x_2 \leq 12, \quad x_1, x_2 \geq 0.$
   Answer: $x_1 = 3,\ x_2 = 0, \quad \text{Max } Z = 21.$

   (b) Maximize $Z = 3x_1 + 2x_2$ s.t. $2x_1 + x_2 \leq 40, \quad x_1 + x_2 \leq 24, \quad 2x_1 + 3x_2 \leq 60, \quad x_1, x_2 \geq 0.$
   Answer: $x_1 = 16,\ x_2 = 8, \quad \text{Max } Z = 64.$

   (c) Maximize $Z = 3x_1 + 5x_2$ s.t. $3x_1 + 2x_2 \leq 18, \quad x_1 \leq 4, \quad x_2 \leq 6, \quad x_1, x_2 \geq 0.$
   Answer: $x_1 = 2,\ x_2 = 6, \quad \text{Max } Z = 36.$

3. Max. $Z = 3x_1 + 4x_2$ s.t. $x_1 - x_2 \leq 1, \quad -x_1 + x_2 \leq 2, \quad x_1, x_2 \geq 0.$

   Answer: unbounded.

4. Max. $Z = 8x_1 + 11x_2$ s.t. $3x_1 + x_2 \leq 7, \quad x_1 + 3x_2 \leq 8, \quad x_1, x_2 \geq 0.$

   Answer: $x_1 = 13/8,\ x_2 = 17/8,$ Max $Z = 291/8.$

5. Min. $Z = x_1 + x_2 + 3x_3$ s. t. $3x_1 + 2x_2 + x_3 \leq 3, \quad 2x_1 + x_2 + 2x_3 \leq 2, \quad x_1, x_2, x_3 \geq 0.$

   Answer: $x_1 = 0,\ x_2 = 0,\ x_3 = 0,$ Min $Z = 0.$

6. Max. $Z = 2x_1 + 4x_2 + 3x_3$ s. t. $3x_1 + 4x_2 + 2x_3 \leq 60, \quad 2x_1 + x_2 + 2x_3 \leq 40, \quad x_1 + 3x_2 + 2x_3 \leq 80, \quad x_1, x_2, x_3 \geq 0.$

   Answer: $x_1 = 0,\ x_2 = 20/3,\ x_3 = 50, 3,$ Max $Z = 250/3.$

7. Max. $Z = 5x_1 + 2x_2 + 3x_3 - x_4 + x_5$ s. t. $x_1 + 2x_2 + 2x_3 + x_4 = 8, \quad 3x_1 + 4x_2 + x_3 + x_5 = 7, \quad x_1, x_2, x_3, x_4, x_5 \geq 0.$

   Answer: $x_1 = 0,\ x_2 = 0,\ x_3 = 0,\ x_4 = 4,\ x_5 = 3,$ Max $Z = 15.$

8. Max. $Z = 3x_1 + 2x_2 - 2x_3$ s. t. $x_1 + 2x_2 + 2x_3 \leq 10, \quad 2x_1 + 4x_2 + 3x_3 \leq 15, \quad x_1, x_2, x_3 \geq 0.$

   Answer: $x_1 = 15/2,\ x_2 = 0,\ x_3 = 0,$ Max $Z = 45/2.$

9. Min. $Z = 4x_1 + 8x_2 + 3x_3$ s. t. $x_1 + x_2 \geq 2, \quad 2x_1 + x_3 \geq 5, \quad x_1, x_2, x_3 \geq 0.$

   Answer: $x_1 = 5/2,\ x_2 = 0,\ x_3 = 0,$ Min $Z = 10.$

Solve the following LPP using Big-M method

1. Max. $Z = 3x_1 - x_2$ subject to $2x_1 + x_2 \geq 2, \quad x_1 + 3x_2 \leq 2, \quad x_2 \leq 4, \quad x_1, x_2 \geq 0.$

   Answer: $x_1 = 2,\ x_2 = 0,$ Max. $Z = 6.$

2. Max. $Z = 5x_1 - 2x_2 + 3x_3$ subject to $2x_1 + 2x_2 - x_3 \geq 2, \quad 3x_1 - 4x_2 \leq 3, \quad x_2 + 3x_3 \leq 5, \quad x_1, x_2, x_3 \geq 0.$

   Answer: $x_1 = 23/3,\ x_2 = 5,\ x_3 = 0$ Max. $Z = 85/3.$

3. Max. $Z = 3x_1 + 2x_2 + x_3 + 4x_4$ subject to $4x_1 + 5x_2 + x_3 + 5x_4 = 5, \quad 2x_1 - 3x_2 - 4x_3 + 5x_4 = 7, \quad x_1 + 4x_2 + 5x_3 - 4x_4 = 6, \quad x_1, x_2, x_3, x_4 \geq 0.$

   Answer: No feasible solution.

4. Maximize $Z = -2x_1 - x_2$ subject to $3x_1 + x_2 = 3, \quad 4x_1 + 3x_2 \geq 6, \quad x_1 + 2x_2 \leq 4, \quad x_1, x_2 \geq 0.$

   Answer: $x_1 = \frac{3}{5},\ x_2 = \frac{6}{5},$ Max $Z = -\frac{12}{5}.$

5. Min. $Z = 5x_1 + 6x_2$ subject to $2x_1 + 5x_2 \geq 1500, \quad 3x_1 + x_2 \geq 1200, \quad x_1, x_2 \geq 0.$

   Answer: $x_1 = \frac{4500}{13},\ x_2 = \frac{2100}{13},$ Min $Z = 2700.$

6. Max. $Z = 8x_2$ subject to $x_1 - x_2 \geq 0, \quad 2x_1 + 3x_2 \leq -6,$ x_1, x_2 are unrestricted.

   Answer: $x_1 = -6/5,\ x_2 = -6/5,$ Max$Z = -48/5.$

7. Max. $Z = 4x_1 + 2x_2$ s.t. $3x_1 + x2 \leq 27, \quad x_1 + x_2 \geq 21, \quad x_1, x_2 \geq 0.$

   Answer: $x_1 = 0,\ x_2 = 27,$ Max$Z = 54.$

8. Min. $Z = x_1 + x_2 + 3x_3$ s.t. $3x_1 + 2x_2 + x_3 \leq 3, \quad 2x_1 + x_2 + 2x_3 \geq 3, \quad x_1, x_2, x_3 \geq 0.$

   Answer: $x_1 = 3/4,\ x_2 = 0,\ x_3 = 3/4,$ Min$Z = 3.$

Q. Food A contains 20 units of vitamin X and 40 units of vitamin Y per gram. Food B contains 30 units each of vitamins X and Y. The daily minimum requirements are 900 units of X and 1200 units of Y. Costs: How many grams of each type of food should be consumed so as to minimize the cost if food A costs 60 paise per gram and food B costs 80 paise per gram.

Answer: Food $A = 15$gm, Food $B = 20$gm, Total Cost = Rs. 25.

Q. A finished product must weigh exactly 150 gm. The two raw materials used in manufacturing the product are: A with a cost of Rs. 2 per unit and B with a cost of Rs. 8 per unit. At least 14 units of B and not more than 20 units of A must be used. Each unit of A and B weighs 5 and 10 grams, respectively. How much of each type of raw material should be used for each unit of the final product in order to minimize the cost?

Answer: $x_A = 2, x_B = 14,$ Min Cost = Rs. 116.