Problem Sheet 1
(LPP: Formulation and Graphical Solution)

Formulation of Linear Programming Problems.

Formulate each of the following problems as a Linear Programming Problem (LPP).

1. A furniture dealer deals in two items, tables and chairs. He has Rs. 10000 to invest and storage space for at most 60 pieces. A table costs Rs. 500, and a chair costs Rs. 100. He can sell all the items he buys, earning a profit of Rs. 50 per table and Rs. 15 per chair. Formulate the LPP to maximize profit.

2. A factory produces two products A and B. Each unit of A requires 2 hours moulding, 3 hours grinding and 4 hours polishing. Each unit of B requires 4 hours moulding, 2 hours grinding and 2 hours polishing. The moduling machine can work for 20 hours, the grinding machine for 24 hours, and the polishing machine for 13 hours hours. The profit is Rs. 5 per unit of A and Rs. 3 per unit of B. Formulate the LPP to maximize profit.

3. A dietician prescribes a minimum daily intake of vitamins A, B, and C as 30, 20, and 16 units, respectively. Per gram of two food types, X and Y supply vitamins as follows:

Food	Vitamin A	Vitamin B	Vitamin C
X	7	5	2
Y	2	4	8

   Food X costs Rs. 2 per gram, and food Y costs Rs. 1 per gram. Formulate the LPP to minimize cost.

4. A firm produces products A and B, requiring operations of grinding, turning, assembling, and testing with hours per unit as:

Product	Grinding	Turning	Assembling	Testing
A	1	3	4	5
B	2	1	3	4

   The available capacities of these operations in hours for the given time period are 30, 60, 200, and 200, respectively. Profit on each unit is Rs. 3 for A and Rs. 2 for B. Formulate the LPP to maximize profit.

5. A toy company manufactures two types of dolls, A and B. The plant can produce 2000 dolls per day if only type A is produced. A doll of type B takes twice as long to produce as one of type A. Plastic supply allows the production of 1500 dolls per day (both A and B combined). Type B requires a fancy dress, and only 600 pieces of this are available per day. If the company makes profit of Rs. 3 per doll on doll A and Rs. 5 per doll on doll B, formulate the problem as an LPP to maximize the profit.

6. A furniture firm manufactures chairs and tables using machines A, B, and C. Production of one chair requires 2, 1, and 1 hours on machines A, B, and C, respectively. Each table requires 1, 1, and 3 hours, respectively. The profit realized by selling one chair is Rs. 30, while for a table, the figure is Rs. 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours, and on machine C is 90 hours. Formulate the LPP.

7. A diet must contain at least 4000 units of carbohydrates, 500 units of fat, and 300 units of protein. Food A costs Rs. 2 per unit, and food B costs Rs. 4 per unit. Nutritional values are:

Food	Carbohydrates	Fat	Protein
A	10	20	15
B	25	10	20

Formulate the LPP to find the minimized cost for a diet that consists of a mixture of these two foods and also meets the minimum nutrition requirement.

8. A lamp manufacturer produces two types of lamps A and B. Lamp A requires 2 hours of cutter time and 1 hour of finisher time. Lamp B requires 1 hour of cutter time and 2 hours of finisher time. Available time per month is 104 cutter hours and 76 finisher hours. Profit is Rs. 6 for lamp A and Rs. 11 for lamp B. Assuming that he can sell all that he produces, how many of each type of lamp should he manufacture per month to obtain the best return? Formulate an LPP for this problem.

9. An oil refinery uses two blending processes with input-output per production run:

Process	**Inputs**		**Outputs**	
	Crude A	Crude B	Gasoline X	Gasoline Y
1	6	4	6	9
2	5	6	5	5

The maximum amounts available of crudes A and B are 500 units and 400 units, respectively. Market demand shows that at least 300 units of gasoline X and 260 units of gasoline Y must be produced. The profits per production run for processes 1 and 2 are Rs. 40 and Rs. 50, respectively. Formulate the LPP for maximizing the profit.

10. A factory produces products A and B. To manufacture one unit of product A, a machine has to work 3/2 hours, and a craftsman has to work for 2 hours. To manufacture one unit of product B, the machine has to work 3 hours, and the craftsman has to work for one hour. In a week the factory can avail of 80 hours of machine time and 70 hours of craftsman's time. The profit on the sale of each unit of A and B is Rs. 10 and Rs. 8, respectively. If the manufacturer can sell all the items produced, how many of each should be produced to get the maximum profit per week? Formulate the problem as an LPP.

11. A company manufactures two kinds of leather purses, A and B. A is a high-quality purse, and B is lower quality. The sales of each of these purses, A and B, earn a profit of Rs. 4 and Rs. 3, respectively. Each purse of type A requires twice as much time as a purse of type B, and if all purses are of type B, the company could make 1000 purses per day. The supply of leather is sufficient for only 800 purses per day (both A and B combined). Purse A requires a fancy buckle, and only 400 buckles per day are available. There are only 700 buckles available for purse B. What should be the daily production of each type of purse to get the maximum profit? Formulate the problem as an LPP.

12. A firm produces cloth types A, B, and C using red, green, and blue wool. One unit length of

    - cloth A requires 2 yards of red wool and 3 yards of blue wool.
    - cloth B requires 3 yards of red wool, 2 yards of green wool, and 2 yards of blue wool.
    - cloth C requires 5 yards of green wool and 4 yards of blue wool.

    The firm has only a stock of 8 yards of red wool, 10 yards of green wool, and 15 yards of blue wool. It is assumed that the income obtained from one unit length of type A cloth is Rs. 3.00, of type B cloth is Rs. 5.00, and of type C cloth is Rs. 4.00. Formulate this problem as a linear programming model to maximize the income from the finished cloth.

13. A firm manufactures products A, B, and C with profits of Rs. 3, Rs. 2, and Rs. 4, respectively. The firm has 2 machines, the processing time of which in minutes is

Machine	A	B	C
G	4	3	5
H	2	2	4

Machine G has 2000 minutes, and machine H has 2500 minutes. The firm must manufacture at least 100 A, 200 B, 50 C, and at most 150 A. Set up a linear programming problem to maximize profit.

14. A company employs inspectors of Grade I and Grade II. At least 2000 pieces must be inspected in an 8-hour day. Grade I inspectors can inspect 50 pieces/hour with 97% accuracy, and Grade II inspectors can check 40 pieces/hour with 95% accuracy. The wage rate of Grade I inspector is Rs. 4.50/hour and that of Grade II inspector is Rs. 2.50/hour, respectively. Each time an error is made by an inspector, the cost to the company is one rupee. For the inspection job, the company has available 10 grade I and 5 grade II inspectors. Formulate the problem to minimize the total cost of inspection.

Solution of LPP using the Graphical Method.

Solve graphically the following LPP:

(a) Maximize $Z = x_1 + x_2$, subject to $x_1 + 2x_2 \leq 2000;\ -x_1 + x_2 \leq 1500;\ x_2 \leq 600;\ x_1, x_2 \geq 0$.
Answer: Infinite number of optimal solutions. One is $x_1 = 1000,\ x_2 = 500,\ Z = 1500$.

(b) Max. $Z = 8x_1 + 7x_2$, s.t. $3x_1 + x_2 \leq 66000;\ x_1 + x_2 \leq 45000;\ x_1 \leq 20000;\ x_2 \leq 40000;\ x_1, x_2 \geq 0$.
Answer: $x_1 = 10500,\ x_2 = 34500,\ Z = 325500$.

(c) Max. $Z = 5x_1 + 7x_2$, s.t. $x_1 + x_2 \leq 4;\ 3x_1 + 8x_2 \leq 24;\ 10x_1 + 7x_2 \leq 35;\ x_1, x_2 \geq 0$.
Answer: $x_1 = 1.6,\ x_2 = 2.4,\ Z = 24.8$.

(d) Minimize $Z = 3x_1 + 5x_2$, s.t. $-3x_1 + 4x_2 \leq 12;\ 2x_1 - x_2 \geq -2;\ 2x_1 + 3x_2 \geq 12;\ x_1 \leq 4;\ x_2 \geq 2;\ x_1, x_2 \geq 0$.
Answer: $x_1 = 3,\ x_2 = 2;\ Z = 19$.

(e) Max. $Z = 3x_1 + 4x_2$, s.t. $x_1 - x_2 \leq -1;\ -x_1 + x_2 \leq 0;\ x_1, x_2 \geq 0$.
Answer: No feasible solution.

(f) Min. $Z = x_1 + x_2$, s.t. $5x_1 + 10x_2 \leq 50;\ x_1 + x_2 \geq 1;\ x_2 \leq 4;\ x_1, x_2 \geq 0$.
Answer: Infinite number of optimal solutions.

(g) Max. $Z = 0.75x_1 + x_2$, s.t. $x_1 - x_2 \geq 0;\ -0.5x_1 + x_2 \leq 1;\ x_1, x_2 \geq 0$.

(h) Max. $Z = 3x_1 + 4x_2$, s.t. $5x_1 + 4x_2 \leq 200;\ 3x_1 + 5x_2 \leq 150;\ 5x_1 + 4x_2 \geq 100;\ 8x_1 + 4x_2 \geq 80;\ x_1, x_2 \geq 0$.
Answer: $x_1 = \frac{100}{3},\ x_2 = 10;\ Z = 140$.

(i) Max. $Z = x_1 + \frac{1}{2}x_2;\ 3x_1 + 2x_2 \leq 12;\ 5x_1 \leq 10;\ x_1 + x_2 \geq 8;\ -x_1 + x_2 \geq 4;\ x_1, x_2 \geq 0$.
Answer: No feasible solution.

(j) Min. $Z = 2x_1 - 10x_2$, s.t. $x_1 - x_2 \geq 0;\ x_1 - 5x_2 \geq -5;\ x_1, x_2 \geq 0$.
Answer: $x_1 = \frac{5}{4},\ x_2 = \frac{5}{4};\ Z = -10$.

(k) Max. $Z = 3x_1 - 2x_2$, s.t. $x_1 + x_2 \leq 1;\ 2x_1 + 2x_2 \geq 4;\ x_1, x_2 \geq 0$.
Answer: No solution.

(l) Max. $Z = x_1 + x_2$, s.t. $x_1 - x_2 \geq 0;\ -3x_1 + x_2 \geq 3;\ x_1, x_2 \geq 0$.
Answer: No feasible solution.

(m) Min. $Z = -x_1 + 2x_2$, s.t. $-x_1 + 3x_2 \leq 10;\ x_1 + x_2 \leq 6;\ x_1 - x_2 \leq 2;\ x_1, x_2 \geq 0$.
Answer: $x_1 = 2,\ x_2 = 0;\ Z = -2$.

(n) A soft drink plant has two bottling machines, A and B, producing 8-ounce and 16-ounce bottles. The following data is available:

Machine	8 Ounce	16 Ounce
A	100/minute	40/minute
B	60/minute	75/minute

The machines can operate 8 hours per day and 5 days per week. Weekly production cannot exceed 300000 ounces, and the market can absorb 25000 eight-ounce bottles and 7000 sixteen-ounce bottles per week. Profit is 15 paise per 8-ounce bottle and 25 paise per 16-ounce bottle. The planner wishes to maximize his profit subject to all the production and marketing restrictions. Formulate the problem as an LPP and solve graphically.

Answer:

The solution is $x_1 = 25000,\ x_2 = 6250$ Maximum Profit = Rs. 5312.50.

(o) A company produces refrigerators and coolers. Refrigerators are produced in Department I and coolers produced in Department II. The company's two products are produced and sold on a weekly basis. The weekly production cannot exceed 25 refrigerators in Department I and 35 coolers in Department II because of the limited available facilities in these two departments. The company regularly employs a total of 60 workers in the two departments. A refrigerator requires 2 man-weeks of labor, while a cooler requires 1 man-week of labor. A refrigerator contributes a profit of Rs. 60, and a cooler contributes a profit of Rs. 40. How many units of refrigerators and coolers should the company produce to realize maximum profit?

Answer: Refrigerators = 12.5, Coolers = 35 Maximum Profit = Rs. 2150.