Operational Research

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- Graphial method
- Simplex method
- Dual simplex method
- Revised simplex
- Big m method

Iterative method

Graphical method

① mini $Z = 20x_1 + 10x_2$

St $x_1 + 2x_2 \le 40$

$3x_1 + x_2 \ge 30$

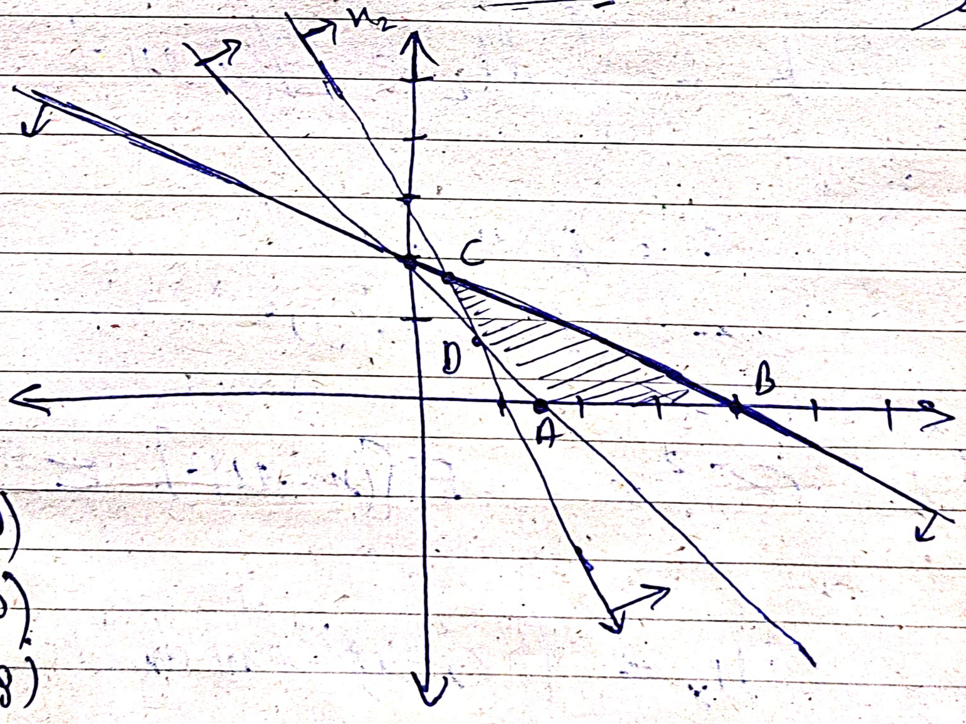
$4x_1 + 3x_2 \ge 60$

$x_1, x_2 \ge 0$

$\frac{3}{60} \; \frac{60}{3} = 20$

$\frac{30}{2}$ = 15

Sol



A (15,0)
B (40,0)
C (4,18)
D (6,12)

$Z_A =$

$Z_B =$

$Z_C =$

$Z_D =$

Graphical method

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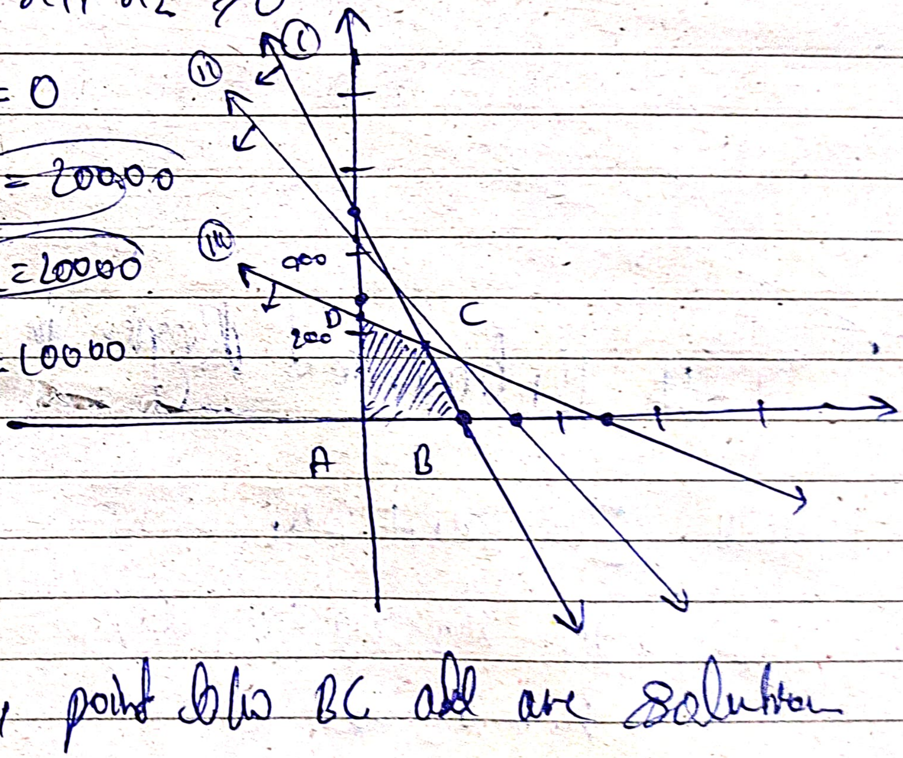
→ unique optimal solution
→ an infinite no. of solution
→ an unbounded solution
→ No solution

$Z = 100x_1 + 40x_2$

s.t.
$5x_1 + 2x_2 \leq 1000$ — (i)
$3x_1 + 2x_2 \leq 900$ — (ii)
$x_1 + 2x_2 \leq 500$ — (iii)
$x_1, x_2 \geq 0$

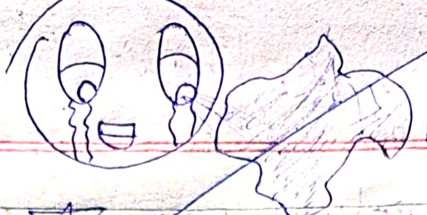
A $(0,0)$ $Z_A = 0$
B $(200,0)$ $Z_B = 20000$
C $(125,187)$ $Z_C = 20000$
D $(0,250)$ $Z_D = 10000$



$Z_B = Z_C$

line BC, point b/w BC all are solution

max value of z occurs at two vertices B C
∴ ∞ no. of points b/w line joining B C,
it gives the same max value of z.
Thus ∞ no. of solution for LPP.



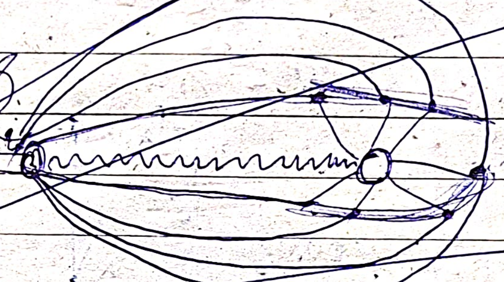
Devops and
OS
Nowdays when marking

~~Who will do what group wise ??~~
~~Bahut Zyada hai ... maximum 5 students~~
~~Lab marking solve ... questions~~

Mai Ho Gian

Communication

~~SYLLABUS~~



~~Raft~~

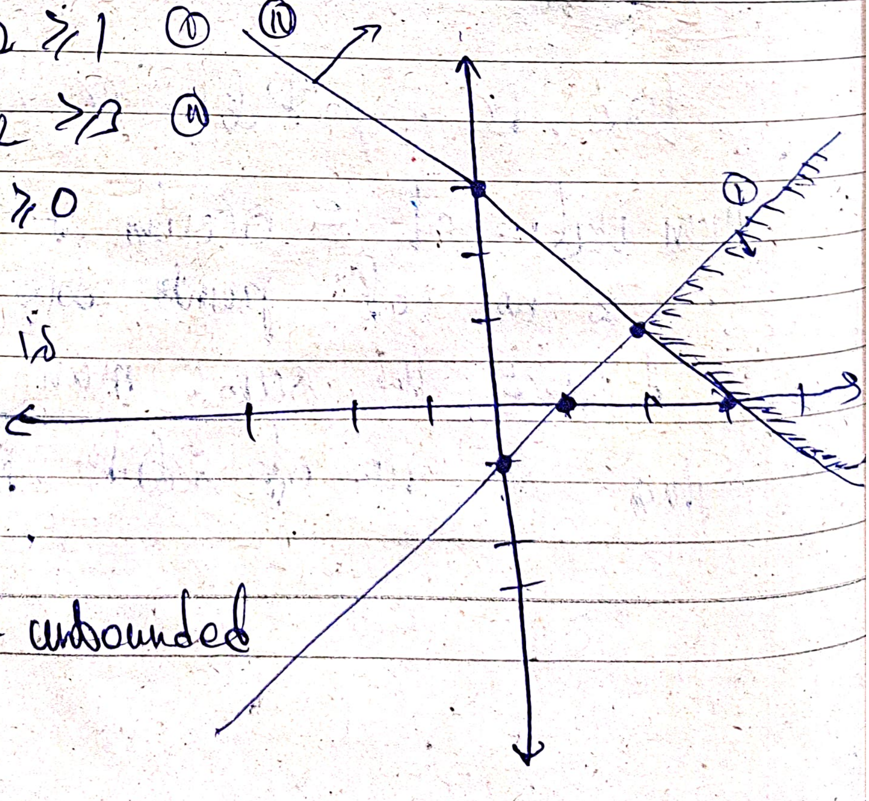
Unbounded Region in Graphical method

max $Z = 3x_1 + 2x_2$

s.t. $x_1 - x_2 \geq 1$ (i)

$x_1 + x_2 \geq 3$ (ii)

$x_1, x_2 \geq 0$



Feasible region is unbounded &

max occurs at ∞.

Hence, problem has unbounded solution

No Feasible Region

max $Z = 3x_1 + 2x_2$

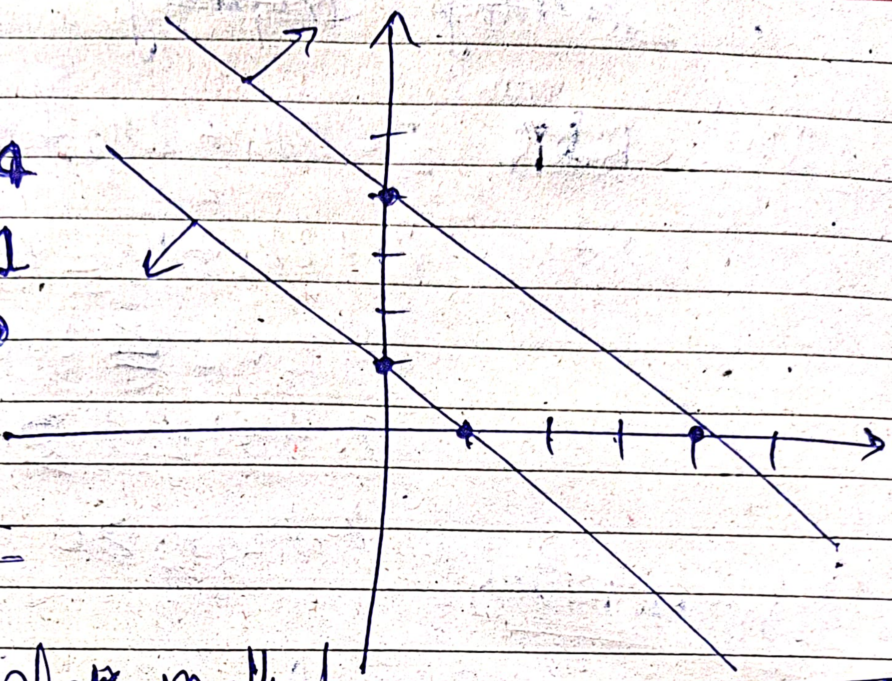
s.t. $x_1 + x_2 \geq 4$

$x_1 + x_2 \leq 1$

$x_1, x_2 \geq 0$

$\Downarrow$

no solution



Simplex method

General linear p.p. (GLpp) — Two decision Variable

① max $Z = 3x_1 + 2x_2$

st $x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

easily solve it by graphical method

But if we have more than two decision variable, then simplex method required to solve that problem.

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~~Centralized Computing.~~

Que ~~max~~ min $Z = 3x_1 + 2x_2 + 5x_3$.

St. $x_1 + 2x_2 + x_3 \le 430$.

$3x_1 + 2x_3 \ge 260$

$x_1 + 4x_2 + x_3 \ge 420$.

$x_1, x_2, x_3 \ge 0$

Simplex method

General form to Standard form

(I) Obj. function → If it is minimized then, changed it into [maximized] function

$$\boxed{\max Z' = -\min Z}$$

(II) check all decision variables are ≥ 0.

$\max Z' = -3x_1 - 2x_2 - 5x_3$.

~~St.~~ $\cancel{-x_1 - 2x_2 - x_3 \ge -430}$

$\cancel{+3x_1 + 2x_3 \ge 260}$

$\cancel{-x_1 - 4x_2 - x_3 > 420}$

$\cancel{x_1, x_2, x_3 \ge 0}$

~~$x_1, x_2, x_3 <$~~

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\# if d.v. are unrestricted.

$2x_1 + x_2 \leq 4 \qquad x_1 \geq 0$ & x_2 is unrestricted

$2x_1 + (x_2' - x_2'') \leq 4 \qquad x_1, x_2', x_2'' \geq 0.$

(iii). Introduce slack or surplus variable to convert inequality constraints into equation.

max $Z = 3x_1 + 2x_2$

s.t. $x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0.$

max $Z = 3x_1 + 2x_2 + 0S_1 + 0S_2$

s.t. $x_1 + x_2 + S_1 = 4$

$x_1 - x_2 + S_2 = 2$

S_1 & S_2 are slack variable with cost zero

max $Z = 3x_1 + 2x_2 + 5x_3 + 0S_1 + 0S_2 - 0S_3$

s.t. $x_1 + 2x_2 + x_3 + S_1 = 430$

$3x_1 + 2x_3 + S_2 = 260$

$x_1 + 4x_2 + x_3 - S_3 = 420.$

S_1 & S_2 are slack variable and S_3 is surplus variable with cost zero.

* $\leq$ add slack variable

$\geq$ subtract surplus variable

(iv) Check whether all the b_i $(i=1,2,3,\ldots n)$ are +ve or not. If any b_i is -ve then multiply the inequation of constraints by (-1).

Ques (1). max $Z = 3x_1 + 2x_2 + 5x_3$

s.t. $x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$

$x_1 + 4x_2 \le 420$

$x_1, x_2, x_3 \ge 0$.

SF

max $Z = 3x_1 + 2x_2 + 5x_3 + 0S_1 + 0S_2 + 0S_3$.

$x_1 + 2x_2 + x_3 + S_1 = 430$

$3x_1 + 2x_3 + S_2 = 460$

$x_1 + 4x_2 + S_3 = 420$

$x_1, x_2, x_3 \ge 0$

$x_1, x_2, x_3, S_1, S_2, S_3 \ge 0$.

Simplex algo

(1) GF → SF

(2) Write initial basic feasible solution

(3) Write standard form LPP into matrix form

(4) Construct initial simplex table

(5) Calculate value of $Z_j - C_j$ and check basic feasible solution for optimality.

$$\boxed{Z_j - C_j = C_B X_j - C_j}$$

$x_1 = x_2 = x_3 = 0, \quad S_1 = 430$

$S_2 = 460$

$S_3 = 420.$

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$$AX = B.$$

$$\left(\begin{array}{cccccc} x_1 & x_2 & x_3 & S_1 & S_2 & S_3 \\ 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} 430 \\ 460 \\ 420 \end{pmatrix}$$

Cost of object fn

basic variable column

outgoing vector

		C_j	3	2	5	0	0	0	min ratio x_B/x_3
C_B	B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	430/1 = 430
0	S_1	430	1	2	1	1	0	0	
0	S_2	460	3	0	$\boxed{2}$	0	1	0	460/2 = 230
0	S_3	420	1	4	0	0	0	1	420/0
$Z_j - C_j$			−3	−2	−5 ↑	0	0	0	

incoming vector

key row

$$\boxed{Z_j - C_j = C_B X_j - C_j}$$

(i) If all $(Z_j - C_j) \geq 0$, the optimal solution will obtained.

(ii) If atleast one $(Z_j - C_j)$ is −ve then indicate by an arrow and this column is called key column.

$Z_1 - C_1 = C_B X_1 - C_1$

$= (0\times1 + 0\times3 + 0\times1) - 3 = -3.$

$Z_2 - C_2 = (0 \quad) - 2 = -2$

(iii) If more than one $(Z_j - C_j)$ is −ve the choose the most −ve of them & this column is called key column.

⑥ Calculate minim ratio.

$$\boxed{\text{min ratio} = \frac{X_B}{x_k}}$$

x_k → key column ≥ 0

First iteration

row column

$x_2 \geq 0$

C_B	B	C_j / X_B	3 / x_1	2 / x_2	5 / x_3	0 / s_1	0 / s_2	0 / s_3	min ratio X_B/x_2
0	s_1	200	$-1/2$	$\boxed{2}$	0	1	$-1/2$	0	$200/2 = \boxed{100}$
5	x_3	230	$3/2$	0	1	0	$1/2$	0	$230/0 = —$
0	s_3	420	1	4	0	0	0	1	$420/4 = \boxed{105}$
$Z_j - C_j$			$9/2$	-2	0	0	$5/2$	0	

outgoing vector

key row

Calculate for above table,

$$\frac{430\times2-460\times1}{2} = \frac{860-460}{2} = \frac{400}{2} = 200$$

$\cancel{420\times2} \neq$ $\frac{420\times2-460\times0}{2} = 420$

$$\frac{1\times2-0\times3}{2} = 1$$

$$\frac{1\times2-3\times1}{2} = \frac{2-3}{2} = -\frac{1}{2}$$

$$\frac{2\times2-0\times1}{2} = 2$$

$$\frac{4\times2-0\times0}{2} = 4$$

$$\frac{2\times0-1\times1}{2} = -\frac{1}{2}$$

$$\frac{2\times0-0\times1}{2} = 0$$

$Z_j - C_j = C_B X_j - C_j$

$Z_3 - C_3 = C_B X_3 - C_3$

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$= 0 + 5 \times 1 + 0 - 5$

$= 0.$

$Z_1 - C_1 = C_B X_1 - C_1$

$= (0 \times \cancel{-1/2} + 5 \times 1/2 + 0 \times 1) - 3.$

$= \frac{15}{2} - 3.$

$= 9/2$

$Z_2 - C_2 = C_B X_2 - C_2$

$= 0 - 2 = -2$

C_B	B	C_j / X_B	3 / x_1	2 / x_2	5 / x_3	0 / S_1	0 / S_2	0 / S_3
2	x_2	100	$-1/4$	1	0	$1/2$	$-1/4$	0
5	x_3	230	$3/2$	0	1	0	$1/2$	0
0	S_3	20	2	0	0	-2	1	1
$Z_j - C_j$			4	0	0	1	2	0

key element se divide karna hai

$R_3' = R_3 - 4R_1$

$-\frac{1}{2} + \frac{15}{2} - 3$

4

420	1	4	0	0	0	1
$-$400	$+$-1	$-$4	$-$0	$-$2	$+$-1	$-$0
20	2	0	0	-2	1	1

$-\frac{1}{2} + \frac{5}{2} = 2$

Since all $Z_j - C_j \geq 0$, the solution is optimum & given by $x_2 = 100$, $x_3 = 230$, $x_1 = 0$.

$\therefore$ Max $Z = \sum C_B X_B$.

$= 2 \times 100 + 5 \times 230$

$= 200 + 1150.$

$= 1350$

Ans (8) minimization:

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$$\text{Min } Z = x_1 - 3x_2 + 2x_3.$$

S.t.
$$3x_1 - x_2 + 2x_3 \le 7$$
$$-2x_1 + 4x_2 \le 12$$
$$-4x_1 + 3x_2 + 8x_3 \le 10$$
$$x_1, x_2, x_3 \ge 0$$

$$\text{Max } Z' = -x_1 + 3x_2 - 2x_3$$

$$3x_1 - x_2 + 2x_3 + S_1 = 7$$
$$-2x_1 + 4x_2 + S_2 = 12$$
$$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$$

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 \end{array}$$
$$\underbrace{\begin{pmatrix} 3 & -1 & 2 & 1 & 0 & 0 \\ -2 & 4 & 0 & 0 & 1 & 0 \\ -4 & 3 & 8 & 0 & 0 & 1 \end{pmatrix}}_{3\times 6} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}}_{6\times 1} = \begin{pmatrix} 7 \\ 12 \\ 10 \end{pmatrix}$$

Initial basic feasible solution,

$x_1 = x_2 = x_3 = 0$, $S_1 = 7$, $S_2 = 12$, $S_3 = 10$.

key element

		C_j	-1	3	-2	0	0	0	Min ratio x_B/x_2 ($x_2 > 0$)
C_B	B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	
0	S_1	7	3	-1	2	1	0	0	—
0	S_2	12	-2	4	0	0	1	0	3
0	S_3	10	-4	3	8	0	0	1	10/3
$Z_j - C_j$			+1	-3	+2	0	0	0	

outgoing vector

key column / incoming vector

$Z_j - C_j = C_B X_j - C_j$

Row operation best hota hai

$R_2' \to R_2/4$

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Incoming vector $R_1' \to R_1 + R_2'$

$x_1 \geq 0$

min ratio $\frac{X_B}{x_1}$

1st iteration

Outgoing vector

		C_j	-1	3	-2	0	0	0	
C_B	B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	
0	S_1	10	5/2	0	2	1	1/4	0	← 4
3	x_2	3	-1/2	1	0	0	1/4	0	—
0	S_3	1	-5/2	0	8	0	-3/4	0	—
$Z_j - C_j$			-1/2 ↑	0	2	0	3/4	0	

$R_3' = R_3 - 3R_2$

10	-4	3	8	0	0	1
9	-3/2	3	0	0	3/4	0
1	-5/2	0	8	0	-3/4	0

$-4 + 3/2$

$-5/2$

$Z_1 - C_1 = X_1 C_B - C_1$
$= 0 - 3/2 + 0 - (-1) = 1 - 3/2 = -1/2$

$Z_2 - C_2 = X_2 C_B - C_2 = 0 + 3 + 0 - 3 = 0$

$Z_3 - C_3 = X_3 C_B - C_3 = 0 + 0 + 0 - (-2) = 2$

$\frac{2}{5} + 2 - (-1)$ $\quad \frac{-4+6}{5}$ $\quad \frac{1}{4} + \frac{1}{20} = \frac{6}{20}$

2nd iteration

		C_j	-1	3	-2	0	0	0
C_B	B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
-1	x_1	4	1	0	4/5	2/5	1/10	0
3	x_2	5	0	1	2/5	1/5	3/10	0
0	S_3	11	0	0	10	1	-1/2	0
			0	0	11/5	1/5	4/5	0

$R_1'' \to \frac{2}{5}R_1'$ $\qquad R_3'' \to R_3' + \frac{5}{2}R_1''$

$R_2'' \to \frac{1}{2}R_1'' + R_2'$

Since all $Z_j - C_j \geq 0$, then solution is optimum

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The optimal solution is given by

$$\max Z' = C_B X_B$$

$$= -9 + 3 \times 5 + 0 \times 11$$

$$= -9 + 15$$

$$= 11$$

$$\text{Min } Z = -\text{Max}(Z')$$

$$\therefore \boxed{\text{Min } Z = -11}$$

$x_1 = 9, \; x_2 = 5, \; x_3 = 0$

Degeneracy in Simplex method

Tie for minimum Ratio.

Ques ① $\max Z = x_1 + 2x_2 + x_3$

S.t.

$$2x_1 + x_2 - x_3 \leq 2$$

$$-2x_1 + x_2 - 5x_3 \geq -6$$

$$4x_1 + x_2 + x_3 \leq 6$$

$$x_1, x_2, x_3 \geq 0$$

Solⁿ →

$$\max Z = x_1 + 2x_2 + x_3 + 0S_1 + 0S_2 + 0S_3$$

$$2x_1 + x_2 - x_3 + S_1 = 2$$

$$-2x_1 + x_2 - 5x_3 - S_2 = -6 \Rightarrow 2x_1 - x_2 + 5x_3 + S_2 = 6$$

$$4x_1 + x_2 + x_3 + S_3 = 6$$

$$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$$

Initial basic feasible solution,

$x_1 = x_2 = x_3 = 0, \quad S_1 = 2, S_2 = 6, S_3 = 6.$

$$\begin{pmatrix} 2 & 1 & -1 & 1 & 0 & 0 \\ 2 & -1 & 5 & 0 & 1 & 0 \\ 4 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 6 \end{pmatrix}$$

		C_j	1	2	1	0	0	0	min ratio
C_B	B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	$x_B/x_2,\ x_2 > 0$
0	S_1	2	2	1	1	1	0	0	2
0	S_2	6	2	$-1 \to 0$	5	0	1	0	—
0	S_3	6	4	$1 \to 0$	1	0	0	1	6
$Z_j - C_j$			-1	$-2 \uparrow$	-1	0	0	0	

$Z_j - C_j = C_B x_j - C_j$

$Z_1 - C_1 = C_B x_1 - C_1$

Note:- Degeneracy in LPP may arise :-

- → at the initial stages.
- → at any subsequent iteration stage

For finding min ratio. → only for +ed rows.

$$= \left(\frac{\text{Elements of first column of unit matrix}}{\text{Corresponding element of key column}} \right).$$

Incoming vector

	C_j		1	2	1	0	0	0	min ratio x_B/x_3, $x_3>0$	x_2/x_3	s_2/x_3
C_B	B	X_B	x_1	x_2	x_3	s_1	s_2	s_3			
R_1' 2	x_2	2	2	1	-1	1	0	0	—	×	∝
R_2' 0	s_2	8	4	0	4	1	1	0	$8/4 = 2$	$0/4=0$	$1/4$
R_3' 0	s_3	4	2	0	2	-1	0	1	$4/2 = 2$	$0/2=0$	$0/2=0$
$Z_j - C_j$			3	0	-3 ↑	2	0	0			

$R_2' = R_1' + R_2$

$R_3' = R_3 - R_1'$

$$\begin{array}{ccc} x_2 & s_2 & s_3 \end{array}$$
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix}$$

x_2 — first column ×; second col

Checks

$Z_j - C_j = C_B X_j - C_j$

		C_j	1	2	1	0	0	0
C_B	B	X_B	x_1	x_2	x_3	s_1	s_2	s_3
R_1'' 2	x_2	4	3	1	0	$1/2$	0	$1/2$
R_2'' 0	s_2	0	0	0	0	3	1	-2
R_3'' 1	x_3	2	1	0	1	$-1/2$	0	$1/2$
$Z_j - C_j$			6	0	0	$1/2$	0	$3/2$

$R_3'' \longrightarrow \frac{R_3'}{2}$

$-1/2 = -2$

$1+2 = 3$

$R_2'' \longrightarrow -4R_3'' + R_2'$

$R_1'' \longrightarrow R_1' + R_3''$

Since all $Z_j - C_j \geq 0$, the solution is optimum,

$\boxed{x_1 = 0}$ $\boxed{x_2 = 4}$ $\boxed{x_3 = 2}$

$\max Z = 2\times 4 + 0\times 0 + 1\times 2$
$= 10$

Unbounded Solution In Simplex method

(i) Unbounded feasible region & unbounded optimal solution

(ii) Unbounded feasible region but bounded optimal solution

Q1. $\max Z = 2x_1 + x_2$

s.t. $x_1 - x_2 \leq 10$

$2x_1 - x_2 \leq 40$

$x_1, x_2 \geq 0$.

$\max Z = 2x_1 + x_2 + 0s_1 + 0s_2$

$x_1 - x_2 + s_1 = 10$

$2x_1 - x_2 + s_2 = 40$

$x_1, x_2, s_1, s_2 \geq 0$

an initial feasible solution,

$x_1 = 0, x_2 = 0, \quad s_1 = 10, \quad s_2 = 40.$

$$\begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{pmatrix}_{2\times 4} \begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{pmatrix}_{4\times 1} = \begin{pmatrix} 10 \\ 40 \end{pmatrix}.$$

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		C_j	2	1	0	0	min x_B/x_1
C_B	B	X_B	x_1	x_2	S_1	S_2	
0	S_1	10	1	−1	1	0	10
0	S_2	40	2	−1	0	1	20
$Z_j - C_j$		=	−2↑	−1	0	0	

gnome

IInd iteration

		C_j	2	1	0	0	min x_B/x_2
	C_B	B	X_B	x_1	x_2	S_1	S_2	
R_1'	2	x_1	10	1	−1	1	0	—
R_2'	0	S_2	20	0	1	−2	1	20.
	$Z_j - C_j$		=	0	−3↑	2	0	

$R_1' \longrightarrow R_1$

$R_2' \longrightarrow R_2 - 2R_1'$

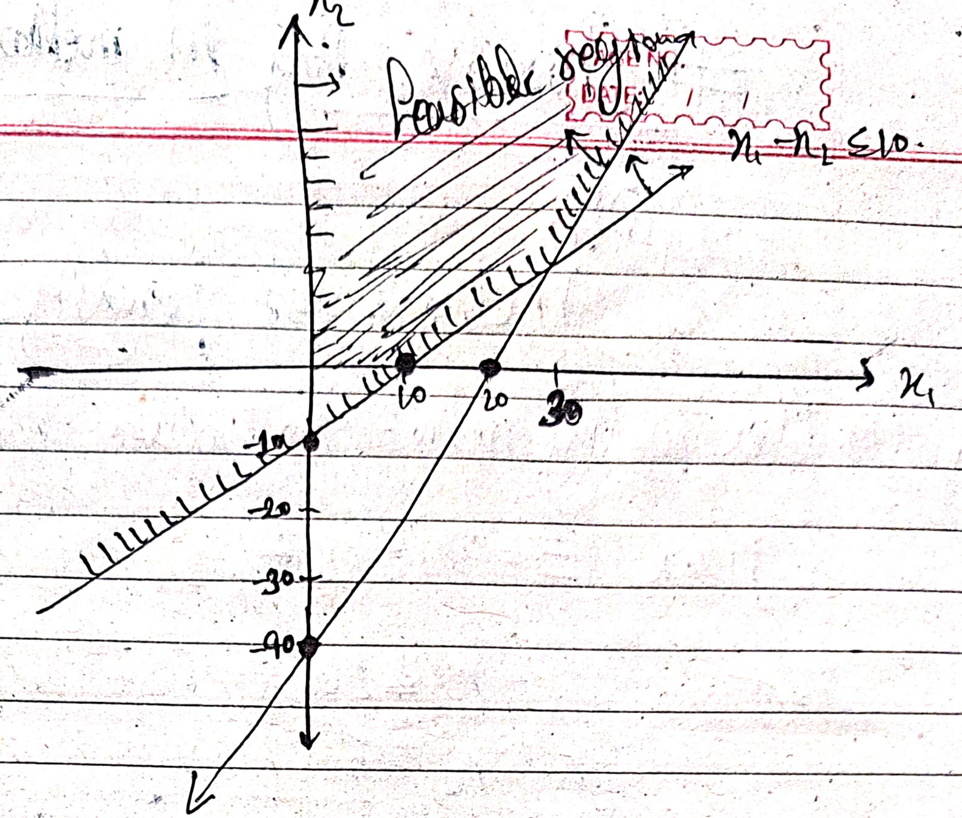
IIIrd iteration

			C_j	2	1	0	0	min ratio
	C_B	B	X_B	x_1	x_2	S_1	S_2	x_B/S_1 ($S_1 \geq 0$)
R_1''	2	x_1	30	1	0	−1	1	—
R_2''	1	x_2	20	0	1	−2	1	—
	$Z_j - C_j$		=	6	0	−4↑	3	

$R_2'' \longrightarrow R_2'$

$R_1'' \longrightarrow R_1' + R_2''$

Since $Z_3 - C_3 = -4 < 0$, the soln is not optimum but all values in key column are negative. which indicates solution is unbounded.



Q no. 2 Max $Z = 6x_1 - 2x_2$

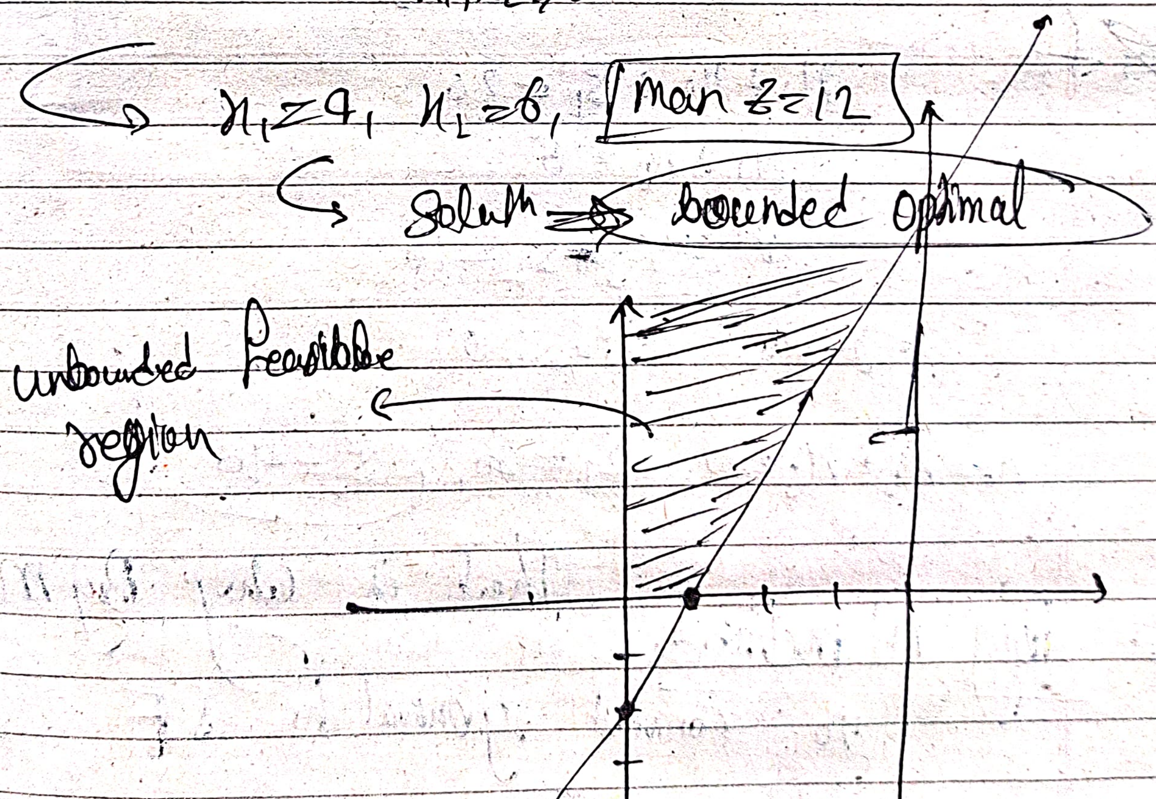
S.t. $2x_1 - x_2 \leq 2$

$x_1 \leq 4$

$x_1, x_2 \geq 0$

$\hookrightarrow$ $x_1 = 4$, $x_2 = 6$, $\boxed{\text{max } Z = 12}$

$\hookrightarrow$ Solutn $\Rightarrow$ bounded optimal



Note:- It is interesting to note that feasible region is not bounded but still the optimal solution is bounded

Charne's Big M method

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Or

method of penality

Que ① Use penality method to solve this

$$\text{Max } Z = x_1 - x_2 + 3x_3$$

S.t.

$$x_1 + x_2 \leq 20$$
$$x_1 + x_3 = 5$$
$$x_2 + x_3 \geq 10$$
$$x_1, x_2, x_3 \geq 0$$

$$\text{max } Z = x_1 - x_2 + 3x_3$$

S.t.

$$x_1 + x_2 + S_1 = 20$$
$$x_1 + x_3 = 5$$
$$x_2 + x_3 - S_2 = 10$$
$$x_1, x_2, x_3, S_1, S_2 \geq 0$$

$$x_1 = x_2 = x_3 = 0, \quad S_1 = 20, \quad S_2 = -10$$

That is why Big M method

In Big M method

To convert general to SF

$\leq$, add slack variable

$\geq$. Subtract a surplus variable & add artificial variable

$=$. add artificial variable.

$\max Z = x_1 - x_2 + 3x_3 + 0S_1 + 0S_2 - MA_1 - MA_2$

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S.t. $x_1 + x_2 + S_1 = 20$

$x_1 + x_3 + A_1 = 5$

$x_2 + x_3 - S_2 + A_2 = 10.$

$x_1, x_2, x_3, S_1, S_2, A_1, A_2 \geq 0.$

$x_1 = x_2 = x_3 = 0, \quad S_1 = 20, \quad A_1 = 5, \quad A_2 = 10.$

$$\begin{array}{ccccccc} x_1 & x_2 & x_3 & S_1 & S_2 & A_1 & A_2 \end{array}$$

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ S_1 \\ S_2 \\ A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 20 \\ 5 \\ 10 \end{pmatrix}$$

M is very large value

So, $-M$ is very very small value

i.e, $-10M < -5M$

incoming vector

		C_j	1	-1	3	0	0	-M	-M	min ratio
C_B	B	X_B	x_1	x_2	x_3	S_1	S_2	A_1	A_2	x_B/x_3, $x_3 > 0$
0	S_1	20	1	1	0	1	0	0	0	—
-M	A_1	5	1	0	1	0	0	1	0	5 ←
-M	A_2	10	0	1	1	0	-1	0	1	10.
	$Z_j - C_j =$		-M-1	-M+1	-2M-3 ↑	0	M	0	0	

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C_B	B	X_B	x_1	x_2	x_3	S_1	S_2	A_1	A_2	min ratio x_B/x_2
		C_j	1	-1	3	0	0	-m	-m	
0	S_1	20	1	1	0	1	0	0	0	20
3	x_3	5	1	0	1	0	0	1	0	—
-m	A_2	5	-1	1	0	0	-1	-1	1	5
$Z_j - C_j =$			m+2	-m+1 ↑	0	0	m	2m+3	0	

$R_3' = R_3 - R_2'$

C_B	B	X_B	x_1	x_2	x_3	S_1	S_2	A_1	A_2
		C_j	1	-1	3	0	0	-m	-m
0	S_1	15	2	0	0	1	1	1	-1
3	x_3	5	1	0	1	0	0	1	0
-1	x_2	5	-1	1	0	0	-1	-1	1
$Z_j - C_j =$			3	0	0	0	1	4+m	-1+m

$R_1'' = R_1' - R_3''$

Soln: modified LM by simplex method contd the any one of three cases may arise.

(i) If no artificial variable appear in basis & the optimality conditions are satisfied, then current soln is optimal basic feasible soln.

(ii) If atleast one arti. vari. is there in basis at zero level and optimality condn satisfied, then current soln is optimal basic feasible solution.

(iii) if atleast one artif variable is appear in basis at +ve level & optimal condn are satisfied, then the original problem has no feasible solution.

Since all $z_j - c_j \geq 0$, the soln is optimum & is given by $x_1 = 0$, $x_2 = 5$, $x_3 = 5$

$$\max Z = C_B X_B = 0 \times 15 + 3 \times 5 + 5 \times (-1)$$
$$= 15 - 5$$
$$= 10$$

Charne's Big-M method (minimization)

Que Min $Z = 12x_1 + 20x_2$

s.t. $6x_1 + 8x_2 \geq 100$

$7x_1 + 12x_2 \geq 120$

$x_1, x_2 \geq 0$

$\max Z' = -(12x_1 + 20x_2)$.

$6x_1 + 8x_2 + A_1 - S_1 = 100$

$7x_1 + 12x_2 + A_2 - S_2 = 120$

$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$.

$x_1 = 0 = x_2 = 0$. $A_1 = 100$, $A_2 = 120$

~~6 8 1 0 -1 0~~ ~~x_1~~

~~7 12 0 1 0 -1~~ ~~x_2 A_1 A_2 S_1~~

matrix form $AX=B$.

$$\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & S_1 & S_2 & A_1 & A_2 \end{array} \\ \begin{pmatrix} 6 & 8 & -1 & 0 & 1 & 0 \\ 7 & 12 & 0 & -1 & 0 & 1 \end{pmatrix}_{2\times 6} \end{array} \begin{pmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \\ A_1 \\ A_2 \end{pmatrix}_{6\times 1} = \begin{pmatrix} 100 \\ 120 \end{pmatrix}$$

$$\max Z' = -12x_1 - 20x_2 + 0S_1 + 0S_2 - MA_1 - MA_2.$$

Initial table

	C_j		-12	-20	0	0	-M	-M	min ratio
C_B	B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	x_B/x_2, $x_2>0$.
-M	A_1	100	6	8	-1	0	1	0	$100/8 = 12.$
-M	A_2	120.	7	12	0	-1	0	1	$120/12 = 10$
$Z_j - C_j$			$-13M+12$	$-20M+20$	M	M	0	0	

x_2 variable enter the basis and A_2 leaves the basis II

	C_j		-12	-20	0	0	-M	-M	min ratio
C_B	B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	x_B/x_1 Above
-M	A_1	20	$4/3$	0	-1	$2/3$	1	$-2/3$	$20/\frac{4}{3} = 15$
-20	x_2	10	$7/12$	1	0	$-1/12$	0	$1/12$	$\frac{120}{7} = 17.1$
$Z_j - C_j$			$\frac{-4M+1}{3}$	0	M	$\frac{-2M+5}{3}$	0	$\frac{5(M+1)}{3}$	

80 $\quad \frac{8\times 7}{12}=\frac{14}{3}$ $\quad$ 8 $\quad$ 0 $\quad -\frac{8}{12}$ $\quad$ 0 $\quad \frac{8}{12}$

80 $\quad \frac{14}{3}$ $\quad$ 8 $\quad$ 0 $\quad -2/3$ $\quad$ 0 $\quad 2/3$.

100 $\quad$ 6 $\quad$ 8 $\quad$ -1 $\quad$ 0 $\quad$ 1 $\quad$ 0

20 $\quad 4/3$ $\quad$ 0 $\quad$ -1 $\quad 2/3$ $\quad$ 1 $\quad -2/3$

$6 - \frac{14}{3}$

$\frac{-4M}{3} - 20\times\frac{7}{12} + 12$ $\qquad -\frac{2}{3}M + 20\times\frac{1}{12}$

$\frac{-4M}{3} \frac{-140+144}{12}$ $\qquad +\frac{2M}{3} - \frac{5}{3} + M$

$= \frac{-4M}{3} + \frac{4}{12}$ 3 $\qquad \frac{5M}{3}$

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		C_j	-12	-20	0	0
C_B	B	x_B	x_1	x_2	S_1	S_2
-12	x_1	15	1	0	$-3/4$	$1/2$
-20	x_2	$5/4$	0	1	$7/16$	$-3/8$
$z_j - c_j$		$=$	0	0	$1/4$	$3/2$

$9 + \frac{\cancel{20}^{5} \cdot 2}{\cancel{16}_4}$

$\frac{36-35}{4}$

$-6 + \frac{\cancel{20}^{5} \times 3}{\cancel{8}_2}$

$\frac{-12+15}{2}$

$20 \quad 4/3 \quad 0 \quad 1 \quad 2/3 \quad 1$

$\frac{3}{4} \times \quad 15 \quad 1 \quad 0 \quad -3/4 \quad \frac{1}{2} \quad 3/4$

$10 \quad 7/12 \quad 1 \quad 0 \quad -1/12 \quad 0$

$7/12 \quad \frac{7 \times \cancel{15}^5}{\cancel{12}_4} \quad \frac{7}{12} \quad 0 \quad \frac{-7}{\cancel{12}_4} \times \frac{3}{4} \quad \frac{7}{24} \quad \frac{7}{16}$

$\frac{40-35}{4} \quad 0 \quad 1 \quad \frac{7}{16} \quad \frac{-\cancel{9}3}{\cancel{24}_8}$

Since all $z_j - c_j \geq 0$, the solut^n is optimum & is given by, $x_1 = 15$, $x_2 = 5/4$

$\text{Max } Z' = -12 \times 15 - \cancel{20}^{5} \times 5/\cancel{4}$
$= -180 - 25$
$= -205$

$\text{Min } Z = - \text{Max } Z'$
$= - -205$

$\text{min } Z = 205$

Big M-method

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For No Feasible Solution

Que Max $Z = 3x_1 + 2x_2$

s.t. $2x_1 + x_2 \le 2$

$3x_1 + 4x_2 \ge 12$

$x_1, x_2 \ge 0$

S.F. $\max Z = 3x_1 + 2x_2 + 0S_1 + 0S_2 - mA_1$

$2x_1 + x_2 + S_1 = 2$

$3x_1 + 4x_2 - S_2 + A_1 = 12$

$x_1, x_2, A_1, S_1, S_2 \ge 0$

Initial feasible sol $x_1 = x_2 = 0$, $S_1 = 2$, $A_1 = 12$

		C_j	3	2	0	0	$-m$	ratio min
C_B	B	X_B	x_1	x_2	S_1	S_2	A_1	x_B/x_2, $x_2 > 0$
0	S_1	2	2	1	1	0	0	2
$-m$	A_1	12	3	4	0	-1	1	3
$Z_j - C_j =$			$-3m-3$	$-4m-2$	0	m	0	

		C_j	3	2	0	0	min ratio
C_B	B	X_B	x_1	x_2	S_1	S_2	x_B/S_2
0	S_1	-1	$5/4$	0	1	$1/4$	8
2	x_2	3	$3/4$	1	0	$-1/4$	0
$Z_j - C_j$			$-3/2$	0	0	$-1/2$	

		C_j	3	2		0	$-M$
C_B	B	X_B	x_1	x_2	[illegible]	S_2	A_1
2	x_2	2	2	1		0	0
$-M$	A_1	[illegible]	-5	0	[illegible]	-1	1
$Z_j - C_j$			$5M+1$	0		M	0

A_i is +ve level & $Z_j - C_j \geq 0$ ⟹ no feasible solⁿ
but LPP possess a pseudo optimal solution

Two phase Simplex method

(1) $\max Z = 5x_1 - 4x_2 + 3x_3$

s.t. $2x_1 + x_2 - 6x_3 = 20$

$6x_1 + 5x_2 + 10x_3 \leq 76$

$8x_1 - 3x_2 + 6x_3 \leq 50$

$x_1, x_2, x_3 \geq 0$

Phase-(1) eliminate artificial variable

construct auxiliary LPP

(i) artificial variable cost → (−1)

other cost → 0

(ii) solve auxiliary LPP by simplex method until ($Z_j - C_j \geq 0$)

→ (a) max $Z^* < 0$ & atleast Artifi var. in optimum basis +ve level

→ (b) max $Z^* = 0$ & atleast Artifi. var in " " zero level

→ (c) max $Z^* = 0$ & no artificial variable appear.

In case ①; given LPP does not possess any feasible solution, whereas in case ②. & ③ we go to phase ②.

Qn.

$$\text{Max } Z = 5x_1 - 4x_2 + 3x_3 + 0S_1 + 0S_2$$

S.t.

$$2x_1 + x_2 - 6x_3 + A_1 = 20.$$

$$6x_1 + 5x_2 + 10x_3 + S_1 = 76$$

$$8x_1 - 3x_2 + 6x_3 + S_2 = 50.$$

$$x_1, x_2, x_3, A_1, S_1, S_2 \geq 0.$$

Initial basic feasible solution,

$x_1 = x_2 = x_3 = 0.$ $\quad A_1 = 20.$

$S_1 = 76$

$S_2 = 50.$

Phase I,

Auxiliary LPP.

$$\text{Max } Z^* = 0x_1 + 0x_2 + 0x_3 + 0S_1 + 0S_2 - A_1$$

St.

$$2x_1 + x_2 - 6x_3 + A_1 = 20.$$

$$6x_1 + 5x_2 + 10x_3 + S_1 = 76$$

$$8x_1 - 3x_2 + 6x_3 + S_2 = 50$$

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & S_1 & S_2 & A_1 \end{array}$$

$$\begin{pmatrix} 2 & 1 & -6 & 0 & 0 & 1 \\ 6 & 5 & 10 & 1 & 0 & 0 \\ 8 & -3 & 6 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ S_1 \\ S_2 \\ A_1 \end{pmatrix} = \begin{pmatrix} 20 \\ 76 \\ 80 \end{pmatrix}$$

		C_j	0	0	0	0	0	-1	min ratio
C_B	B	x_B	x_1	x_2	x_3	S_1	S_2	A_1	x_B/x_1
-1	A_1	20	2	1	-6	0	0	1	10
0	S_1	76	6	5	10	1	0	0	$\frac{76}{6} = 12.5$
0	S_2	50	8	-3	6	0	1	0	$50/8 = 6.2$
$Z_j - C_j$		=	-2↑	-1	6	0	0	0	

		C_j	0	0	0	0	0	-1	min ratio
C_B	B	x_B	x_1	x_2	x_3	S_1	S_2	A_1	x_B/x_2, $x_2 \geq 0$
-1	A_1	15/2	0	7/4	-15/2	0	-1/4	1	$+30/7 = 4.28$
0	S_1	77/2	0	29/4	11/2	1	-3/4	0	$\frac{77 \times 2}{29} = 5.31$
0	x_1	$\frac{25}{4}$ ($\cancel{50/8}$)	1	-3/8	3/4	0	1/8	0	
$Z_j - C_j$			0	-7/4↑	+15/2	0	+1/4	0	

R_1 20 2 1 -6 0 0 1

$-2R_3'$ 25/2 2 $-3/4$ 3/2 0 1/4 0

R_1' 15/2 0 7/4 -15/2 0 -1/4 1

R_2 76 6 5 10 1 0 0

$6R_3'$ 75/2 6 $-9/4$ 9/2 0 3/4 0

R_2' 77/2 0 29/4 11/2 1 -3/4 0

R_1'' $\frac{15}{2} \times \frac{4}{7} = \frac{30}{7}$ 0 1 $-\frac{15}{2} \times \frac{4}{7} = \frac{30}{7}$ 0 $-\frac{1}{4} \times \frac{4}{7} = -\frac{1}{7}$ $\frac{4}{7}$

$\frac{29}{4} R_1''$ $\frac{30}{7} \times \frac{29}{4}$ 0 $\frac{29}{4}$ $-\frac{30}{7} \times \frac{29}{4}$ 0 $-\frac{1}{7} \times \frac{29}{4}$ $\frac{4}{7} \times \frac{29}{4}$

$\frac{435}{14}$ 0 $\frac{29}{4}$ $-\frac{435}{14}$ 0 $-\frac{29}{28}$ $\frac{29}{7}$

$\frac{25}{8} \times 6 = \frac{75}{4}$

$\frac{3}{8} \times 6 = \frac{9}{4}$

$\frac{3}{4} \times 6 = \frac{9}{2}$

$\frac{1}{8} \times 6 = \frac{3}{4}$

$76 - \frac{75}{2}$

		C_j	0	0	0	0	0	-1
C_B	B	X_B	x_1	x_2	x_3	S_1	S_2	A_1
0	x_2	$30/7$	0	1	$-30/7$	0	$-1/7$	$4/7$
0	S_1	$52/7$	0	0	$\frac{-256}{7}$	1	$2/7$	$\frac{-29}{7}$
0	x_1	$\frac{55}{7}$	1	0	$-6/7$	0	$1/14$	$3/14$
$Z_j - C_j$			0	0	0	0	0	+1

R_2' $\;77/2\;$ 0 $\;29/4\;$ $1/2\;$ 1 $\;-7/4\;$ 0

$-\frac{29}{4}R_1''$ $\;\frac{435}{14}\;$ 0 $\;29/4\;$ $-\frac{435}{14}\;$ 0 $\;\frac{-29}{28}\;$ $\frac{29}{7}$

$\frac{52}{7}$ 0 0 $-\frac{256}{14}$ 1 $\frac{2}{7}$ $\frac{-29}{7}$

$-\frac{-21}{28}+\frac{29}{28}=\frac{8}{28}=\frac{2}{7}$

R_3' $\;25/4\;$ 1 $\;-3/8\;$ $3/8\;$ 0 $\;1/8\;$ 0

$+\frac{3}{8}R_1''$ $\;\frac{45}{28}\;$ 0 $\;3/8\;$ $-\frac{45}{28}\;$ 0 $\;\frac{-3}{56}\;$ $3/14$

$\frac{8}{7}\times\frac{3}{8}$ $\frac{3}{14}$

$\frac{55}{7}$ 1 0 $\frac{-6}{7}$ 0 $\frac{1}{14}$ $\frac{3}{14}$

$\because$ all $Z_j - C_j \geq 0$

$\hookrightarrow$ we have reached to optimal solution,

&

max $Z^* = C_B X_B = 0$. & no artif var in basic ?

$\hookrightarrow$ case (iii),

So, we go to phase (II)

phase (II)

Consider final simplex table of phase (I) & Consider actual cost associated with original variables delete the artificial variable A_1 column from table as it is eliminated in phase (I),

		C_j	5	-4	3	0	0
C_B	B	X_B	x_1	x_2	x_3	S_1	S_2
-4	x_2	30/7	0	1	-20/7	0	-1/7
0	S_1	52/7	0	0	$-\frac{256}{7}$	1	2/7
5	x_1	55/7	1	0	-6/7	0	1/14
$Z_j - C_j$			0	0	69/7	0	13/14

$= 4 \times \frac{30}{7} - \frac{6}{7} \times 5 - 3$

$= \frac{120 - 30 - 21}{7}$

$= 69/7$

$+\frac{1}{7} \times 4 + \frac{5}{14} = \frac{8+5}{14} = \frac{13}{14}$

$\because$ all $Z_j - C_j \geq 0$, an optimal feasible solution has been reached. Hence, an optimum feasible solution to given LPP is

$$\boxed{x_1 = \frac{55}{7},\ x_2 = \frac{30}{7},\ x_3 = 0}$$

$$\max Z = -\frac{4 \times 30}{7} + 0 + \frac{5 \times 55}{7}$$

$$= \frac{-120 + 275}{7} = \frac{155}{7}$$

minimize — Two phase simplex method

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Qn $\quad$ Min $Z = x_1 - 2x_2 - 3x_3$.

s.t. $\quad -2x_1 + x_2 + 3x_3 = 2$

$2x_1 + 3x_2 + 4x_3 = 1$

$x_1, x_2, x_3 \geq 0$.

phase (I) $\longrightarrow$ eliminate artificial variable

phase (II) $\longrightarrow$ optimum solution.

Soln

Max $Z = -x_1 + 2x_2 + 3x_3$

s.t. $\quad -2x_1 + x_2 + 3x_3 + A_1 = 2$

$2x_1 + 3x_2 + 4x_3 + A_2 = 1$

$x_1, x_2, A_1, A_2, x_3 \geq 0$

Initial basic feasible solution

$x_1 = x_2 = x_3 = 0 \quad A_1 = 2, \quad A_2 = 1$

phase I

Max $Z^* = 0x_1 + 0x_2 + 0x_3 - A_1 - A_2$

s.t.

$-2x_1 + x_2 + 3x_3 + A_1 = 2$

$+2x_1 + 3x_2 + 4x_3 + A_2 = 1$

C_B	B	C_j / X_B	0 / x_1	0 / x_2	0 / x_3	-1 / A_1	-1 / A_2	min ratio X_B/x_3, $x_3 \geq 0$
-1	A_1	2	-2	1	3	1	0	2/3
-1	A_2	1	2	3	4	0	1	1/4 $\leftarrow$
$Z_j - C_j$			0	-4	-7 $\uparrow$	0	0	

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		C_j	0	0	0		
C_B	B	X_B	x_1	x_2	x_3	A_1	A_2
-1	A_1	5/4	-7/2	-5/4	0	1	-3/4
0	x_3	1/4	1/2	3/4	1	0	1/4
$Z_j - C_j$			7/2	5/4	0	0	3/4

2	-2	1	3	1	0
-3/4	-3/2	-9/4	3	0	3/4
5/4	-7/2	-5/4	0	1	-3/4

Sol max $z^* = -5/4$

max $z^* < 0$ & A_1 is in basis at +ve

$z_j - c_j \geq 0$ → optimum feasible soluh to auxiliary lpp has been attained.

So, original problem does not process any feasible solution.

primal to dual Conversion

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Q no 1

max $Z = x_1 + 2x_2 + x_3$.

S.T. $2x_1 + x_2 - x_3 \le 2$

$-2x_1 + x_2 - 5x_3 \ge -6$

$4x_1 + x_2 + x_3 \le 6$

$x_1, x_2, x_3 \ge 0$.

(i) Convert in Canonical form

min ⟶ min $\ge$

max ⟶ max $\le$

(ii) change object funct

max Z ⟶ min Z $\ge$

min Z ⟶ max Z $\le$

(iii). primal { no. of Variable = n ; no of constraints = m

dual { no. of constraint = n ; no. of variable = m.

(iv) Cost coefficient in object funct of the primal will be RHS constant of the constraint in dual and vice-versa.

(v) For formulating constraint we considered the transpose of matrix.

~~Primal to dual Conversion~~

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Canonical form.

Sol. $\max Z = x_1 + 2x_2 + x_3.$

s.t. $2x_1 + x_2 - x_3 \leq 2.$

$2x_1 - x_2 + 5x_3 \leq 6.$

$4x_1 + x_2 + x_3 \leq 6.$

$x_1, x_2, x_3 \geq 0.$

$$\begin{pmatrix} 2 & 1 & -1 \\ 2 & -1 & 5 \\ 4 & 1 & 1 \end{pmatrix}$$

dual. Let w_1, w_2, w_3 be dual variables

$\min Z = 2w_1 + 6w_2 + 6w_3.$

s.t. $2w_1 + 2w_2 + 4w_2 \geq 1$

$w_1 - w_2 + w_3 \geq 2.$

$-w_1 + 5w_2 + w_3 \geq 1$

$w_1, w_2, w_3 \geq 0$

Que (2) $\max Z = x_1 - x_2 + 3x_3$

s.t. $x_1 + x_2 + x_3 \leq 10$

$2x_1 - x_3 \leq 2$

$2x_1 - 2x_2 + 3x_3 \leq 6$

~~$2x_1 - x_3 \leq 2$~~

$x_1, x_2, x_3 \geq 0.$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & -1 \\ 2 & -2 & 3 \end{pmatrix}$$

dual

$\min Z = 10w_1 + 2w_2 + 6w_3$

s.t. $w_1 + 2w_2 + 2w_3 \geq 1$

$w_1 - 2w_3 \geq -1$

$w_1 - w_2 + 3w_3 \geq 3.$

Ans. 3 $\quad \text{Min } Z = 2x_2 + 5x_3$

S.t. $\quad x_1 + x_2 \geq 2$

$2x_1 + x_2 + 6x_3 \leq 6$

$x_1 - x_2 + 3x_3 = 4$

$x_1, x_2, x_3 \geq 0.$

$x_1 + x_2 \geq 2$

$-2x_1 - x_2 - 6x_3 \geq -6$

$x_1 - x_2 + 3x_3 \geq 4$

$-x_1 + x_2 - 3x_3 \geq -4.$

Cannonical Form

$\text{Min } Z = 2x_2 + 5x_3$

$$\begin{vmatrix} 1 & 1 & 0 \\ -2 & -1 & -6 \\ 1 & -1 & 3 \\ -1 & 1 & -3 \end{vmatrix}$$

dual

w_1, w_2, w_3', w_3''

$\text{max } Z = 2w_1 - 6w_2 + 4w_3' - 4w_3''$

S.t. $\quad w_1 - 2w_2 + w_3' - w_3'' \leq 0$

$w_1 - w_2 - w_3' + w_3'' \leq 2$

$-6w_2 + 3w_3' - 3w_3'' \leq 5.$

$w_3' - w_3'' = w_3$

$\text{max } Z = 2w_1 - 6w_2 + 4w_3.$

S.t. $\quad w_1 - 2w_2 + w_3 \leq 0.$

$w_1 - w_2 - w_3 \leq 2.$

$-6w_2 + 3w_3 \leq 5.$

$w_1, w_2 \geq 0$, w_3 is unrestricted primal to dual

Qn. $\max Z = x + 2y$

S.t. $2x + 3y \geq 4$

$3x + 4y = 5$

$x \geq 0$, y is unrestr

$$\begin{pmatrix} 2 & -3 & 3 \\ 3 & 4 & -4 \\ -3 & -4 & 4 \end{pmatrix}$$

Sol. $y = y' - y''$, $y', y'' \geq 0$.

$\max Z = x + 2y' - 2y''$

$2x + 3y' - 3y'' \geq 4$

$3x + 4y' - 4y'' = 5$.

$x, y', y'' \geq 0$.

Canonical f..

$-2x - 3y' + 3y'' \leq -4$

$3x + 4y' - 4y'' \leq 5$.

$-3x - 4y' + 4y'' \leq -5$

$x, y', y'' \geq 0$

w_1, w_2', w_2''

dual

$\min Z = -4w_1 + 5w_2' - 5w_2''$

S.t. $-2w_1 + 3w_2' - 3w_2'' \geq 1$

$-3w_1 + 4w_2' - 4w_2'' \geq 2$

$3w_1 - 4w_2' + 4w_2'' \geq -2$

$\min Z = -4w_1 + 5w_2$

S.t. $-2w_1 + 3w_2 \geq 1$

$-3w_1 + 4w_2 \geq 2$

$3w_1 - 4w_2 \geq -2$.

$\boxed{-3w_1 + 4w_2 = 2}$

$w_1 \geq 0$, w_2 is unrestricted.

Dual Simplex method

Q. min $Z = 5x_1 + 6x_2$

s.t. $x_1 + x_2 \geq 2$

$4x_1 + x_2 \geq 4$

$x_1, x_2 \geq 0$

Sol.

max $Z' = -5x_1 - 6x_2$

s.t. $-x_1 - x_2 \leq -2$

$-4x_1 - x_2 \leq -4$

max $Z' = -5x_1 - 6x_2 + 0S_1 + 0S_2$

$-x_1 - x_2 + S_1 = -2$

$-4x_1 - x_2 + S_2 = -4$

Initial basis solution

$x_1 = x_2 = 0$. $S_1 = -2$

$S_2 = -4$

		C_j	-5	-6	0	0
C_B	B	X_B	x_1	x_2	S_1	S_2
0	S_1	-2	-1	-1	1	0
0	S_2	-4	-4	-1	0	1
$Z_j - C_j$			5	6	0	0

max i rule $\frac{Z_j - C_j}{S_2}$, $S_2 < 0$

max $\left[\frac{5}{-4}, \frac{6}{-1}\right] = \max[-1.25, -6]$

-1.25

~~all $Z_j - C_j \geq 0$ then current soln is optimal~~ soln

Test for optimality

(a) If all the values of $X_B \geq 0$ & $Z_j - C_j \geq 0$ then current solution is optimal solution, terminate the process

(b) If any $X_B < 0$, then select most negative X_B, & this row is called key row.

(c) Find maximum ratio $= \dfrac{Z_j - C_j}{\text{key row}}$, key row < 0.

& this column is called key column

(d) Find the new soln table

		C_j	-5	-6	0	0	max ratio
C_B	B	X_B	x_1	x_2	S_1	S_2	$\dfrac{Z_j - C_j}{S_1}$, $S_1 < 0$.
0	S_1	-1	0	$-3/4$	1	$1/4$	
-5	x_1	1	1	$+1/4$	0	$-1/4$	$\left\{\dfrac{19/4}{-3/4}, \dfrac{5/4}{3/4}\right\} = \left(\dfrac{19}{3}, \dfrac{5}{3}\right)$
$Z_j - C_j$			0	$19/4$	0	$5/4$	

$1 \quad 1 \quad 1/4 \quad 0 \quad -1/4$

$\frac{1}{3} \quad 0 \quad 1/4 \quad -1/3 \quad 1/4$

$8 + \frac{5}{3} = \frac{29}{3}$

		C_j	-5	-6	0	0
C_B	B	X_B	x_1	x_2	S_1	S_2
0	S_2	[illegible]	0	-4	$2/3$	1
-5	x_1	$2/3$	1	0	$1/3$	0
$Z_j - C_j$			0	6	$-5/3$	0

$1 \quad 1 \quad 1/4 \quad 0 \quad -1/4$

$+\; -1/3 \quad 0 \quad -1/4 \quad 1 \quad 1/4$

$2/3 \quad 1 \quad 0 \quad 1/3 \quad 0.$

		C_j	-5	-6	0	0
C_B	B	X_B	x_1	x_2	S_1	S_2
0	S_2	4	0	3	-4	1
-5	x_1	2	1	1	-1	0
$Z_j - C_j$			0	1	5	0

$\because$ all $Z_j - C_j \geq 0$ & $X_B \geq 0$, the current basic feasible solution is optimum.

The optimal solution is given by

$$x_1 = 2, \quad x_2 = 0$$

$$\max Z' = 0 \times 4 + 2 \times (-5)$$

$$Z' = -10$$

$$\min Z = -\max Z' = -(-10) = 10$$

Operational Research

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SF

$$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

s.t.

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le = \ge b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le = \ge b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le = \ge b_m$$

$$x_1, x_2, \ldots, x_n \ge 0$$

matrix Notation

Find $x_1, x_2, \ldots, x_n$, to optimize

$Z = Cx$ → Price vector

s.t. $Ax \le = \ge b$ → requirement vector

$x \ge 0$ → Null matrix

decision variable vector

Coefficient matrix

(1)

neck → 100   lace → 300

x_1 per day x_2 per day

$\frac{1}{2}$ hour   1 hour.

$$Z = 300x_2 + 100x_1$$

$$\frac{1}{2}x_1 + x_2 \leq 16.$$

$$x_1 + x_2 \leq 24$$

$$x_1, x_2 \geq 0$$

(2)

x_1 → no of bags of T_1 x_2 x_3.

T_1 T_2 T_3

$2C_1$ $3C_3$ $3C_1$ $2C_2$ $2C_3$ $5C_2$ $4C_3$.

$C_1 \rightarrow 20$

$C_2 \rightarrow 25$

$C_3 \rightarrow 30$

$$\boxed{Z = 6x_1 + 10x_2 + 8x_3.}$$

for C_1

$$2x_1 + 3x_2 \leq 20$$

$$2x_2 + 5x_3 \leq 25$$

$$3x_1 + 2x_2 + 4x_3 \leq 30$$

$$x_1, x_2, x_3 \geq 0.$$

Basic Solution

$Ax = b$ → m eqn
→ n variable

$n > m$

$(n-m)$ variable → 0.

ensure remaining m variables $\det(\,) \neq 0$

max no. of basic solution $= {}^{n}C_{m} = \dfrac{n!}{m!(n-m)!}$

$x_1 + x_2 + x_3 = 4$
$2x_1 + x_2 + 3x_3 = 6$

$m = 2$
$n = 3$

$n - m = 1$ variable

$\det \neq 0$

non-degenerate

Case 1

$x_3 = 0$,

$x_1 + x_2 = 4$
$2x_1 + x_2 = 6$ → $x_1 = 2$, $x_2 = 2$

basic soln ⇒ $(2, 2, 0)$

basic variable x_1, x_2
non-basic variable : x_3

Case 2

$x_2 = 0$.

$x_1 + x_3 = 4$
$2x_1 + 3x_3 = 6$ → $x_1 = 6$, $x_3 = -2$.

basic soln $(6, 0, -2)$
but not feasible
→ (-ve)

Non-degenerate Basic Solution

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- → basic soln ⇒ none of basic variable equals zero
- → Contain exactly m non-zero variables & (n−m) zero variables
- → represent a unique vertex in feasible region

eg:- Basic soln ⇒ (2, 2, 0)

$m = 2$

$n = 3$

basic vars ⇒ $x_1 = 2, x_2 = 2$,

$n - m = 1$

Non basic $x_3 = 0$

⇒ Non-degenerate.

Degenerate basic solution

- → BS → at least one basic var equals to zero.
- → still satisfy eqn
- → multiple basic solutions may map to same point.

eg: (0, 4, 0)

Set x_1, x_2 → BV.
but $x_1 = 0$ } ⇒ degenerate basic solution

Existence & Non-Degeneracy Theorem

→ A necessary & suff cond for existence and non-degeneracy of all basic solution of $Ax = b$ is that every set of m column of augmented matrix $[A \mid b]$ is linearly independent.

meaning of theorem

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* Existence of a basic solution ⇒ det of basic matrix $B \neq 0$.

* Non-degeneracy ⇒ all basic variables are non-zero

* If any m columns become linearly dependent, then:
  - → either basic solution does not exists or
  - → degenerate basic solution appears.

Necessity

* If all basic solutions exist and are non-degenerate, then b can replace any column in a basis, maintaining linear independence.

eg:-

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Augmented matrix, $[A|b] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Column 1 = (1,0) } → linearly dependent
Column 3 = (1,0). }

⇓

degeneracy

Sufficiency part

If any set of m columns of $[A|b]$ is linearly independent, then all basic solutions exist and are non-degenerate.

Reason

any selection of m columns from A gives

$$\det(B) \neq 0.$$

$\Rightarrow$ basic solution exist

Since b is not dependent on fewer columns:

$$x_B = B^{-1}b \neq 0.$$

$\Rightarrow$ non-degenerate.

eg:- $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$$[A|b] = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

b is not dependent on any column subset.

set $x_3 = 0$,

$x_1 + x_2 = 2$

$x_2 = 3$

$x_1 = -1$, $x_2 = 3$.

both basic values $\neq 0$

$\Rightarrow$ Non-degenerate basic solution.