

Problem Sheet 3

(Sensitivity Analysis)

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1. A company produces two products, A and B. The unit revenues are \$2 and \$3, respectively. Two raw materials, M_1 and M_2 , used in the manufacture of the two products have daily availabilities of 8 and 18 units, respectively. One unit of A uses 2 units of M_1 and 2 units of M_2 , and 1 unit of B uses 3 units of M_1 and 6 units of M_2 .
 - (a)
 - i. Determine the dual prices of M_1 and M_2 and their feasibility ranges.
 - ii. Suppose that 2 additional units of M_1 can be acquired at the cost of 25 cents per unit. Would you recommend the additional purchase?
 - iii. What is the most the company should pay per unit of M_2 ?
 - iv. If M_2 availability is increased by 3 units, determine the associated optimum revenue.
 - (b)
 - i. Determine the optimality condition for $\frac{c_A}{c_B}$ that will keep the optimum unchanged.
 - ii. Determine the optimality ranges for c_A and c_B , assuming that the other coefficient is kept constant at its present value.
 - iii. If the unit revenues c_A and c_B are changed simultaneously to \$5 and \$4, respectively, determine the new optimum solution.
 - iv. If the changes in (iii) are made one at a time, what can be said about the optimum solution?
2. Wild West produces two types of cowboy hats. A Type 1 hat requires twice as much labor time as a Type 2. If all the available labor time is dedicated to Type 2 alone, the company can produce a total of 400 Type 2 hats a day. The respective market limits for the two types are 150 and 200 hats per day. The revenue is \$8 per Type 1 hat and \$5 per Type 2 hat.
 - (a) Use the graphical solution to determine the number of hats of each type that maximizes revenue.
 - (b) Determine the dual price of the production capacity (in terms of the Type 2 hat) and the range for which it is applicable.
 - (c) If the daily demand limit on the Type 1 hat is decreased to 120, use the dual price to determine the corresponding effect on the optimal revenue.
 - (d) What is the dual price of the market share of the Type 2 hat? By how much can the market share be increased while yielding the computed worth per unit?
- TOYCO Model:** TOYCO uses three operations to assemble three types of toys: trains, trucks, and cars. The daily available times for the three operations are 430, 460, and 420 minutes, respectively, and the revenues per unit of toy train, truck, and car are \$3, \$2, and \$5, respectively. The assembly times per train at the three operations are 1, 3, and 1 minutes, respectively. The corresponding times per truck and per car are (2, 0, 4) and (1, 2, 0) minutes (a zero time indicates that the operation is not used).
3. In the TOYCO model, suppose that the changes D_1 , D_2 , and D_3 , are made simultaneously in the three operations.
 - (a) If the availabilities of operations 1, 2, and 3 are changed to 440, 490, and 400 minutes, respectively, use the simultaneous conditions to show that the current basic solution remains feasible, and determine the change in the optimal revenue by using the optimal dual prices.
 - (b) If the availabilities of the three operations are changed to 460, 440, and 370 minutes, respectively, use the simultaneous conditions to show that the current basic solution is infeasible.
4. Consider the TOYCO model.
 - (a) Suppose that any additional time for operation 1 beyond its current capacity of 430 minutes per day must be done on an overtime basis at \$50 an hour. The hourly cost includes both labor and the operation of the machine. Is it economically advantageous to use overtime with operation 1?

- (b) Suppose that the operator of operation 2 has agreed to work 2 hrs of overtime daily at \$45 an hour. Additionally, the cost of the operation itself is \$10 an hour. What is the net effect of this activity on the daily revenue?
- (c) Is overtime needed for operation 3?
- (d) Suppose that the daily availability of operation 1 is increased to 440 minutes. Any overtime used beyond the current maximum capacity will cost \$40 an hour. Determine the new optimum solution, including the associated net revenue.
- (e) Suppose that the availability of operation 2 is decreased by 15 minutes a day and that the hourly cost of the operation during regular time is \$30. Is it advantageous to decrease the availability of operation 2?
5. A company produces three products, A, B, and C. The sales volume for A is at least 50% of the total sales of all three products. However, the company cannot sell more than 80 units of A per day. The three products use one raw material, of which the maximum daily availability is 240 lb. The usage rates of the raw material are 2 lb per unit of A, 4 lb per unit of B, and 3 lb per unit of C. The unit prices for A, B, and C are \$20, \$50, and \$35, respectively.
- (a) Determine the optimal product mix for the company.
- (b) Determine the dual price of the raw material resource and its allowable range. If available raw material is increased by 120 lb, determine the optimal solution and the change in total revenue using the dual price.
- (c) Use the dual price to determine the effect of changing the maximum demand for product A by ± 10 units.
6. A company that operates 10 hrs a day manufactures three products on three processes. The following table summarizes the data of the problem:

	(Minutes per unit)			
Product	Process 1	Process 2	Process 3	Unit Price
1	10	6	8	\$4.50
2	5	8	10	\$5.00
3	6	9	12	\$4.00

- (a) Determine the optimal product mix.
- (b) Use the dual prices to prioritize the three processes for possible expansion.
- (c) If additional production hours can be allocated, what would be a fair cost per additional hour for each process?