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N-Gram Language Model

$$P(w_N / w_1^{N-1}) = \frac{\text{Count}(w_1^{N-1} w_N)}{\text{Count}(w_1^{N-1})}$$

Example: → start of sentence

← End of sentence

(2)

<s> I am sam </s>

<s> Sam I am </s>

<s> I do not like Sam </s>

Q.1. $P(\text{am} | \text{I})$

Q.2. Prob of sentence <s> I am sam </s> (Bigram)

Q.3. Prob of $P(\text{Sam} | \text{do})$ using add-1 smoothing (Laplacian Smoothing)

$$\textcircled{1} \quad P(\text{am} | \text{I}) = \frac{\text{Count}(\text{I am})}{\text{Count}(\text{I})}$$

$$= \frac{2}{3}$$

N-Gram Language Model

$$P(w_N / w_1^{N-1}) = \frac{\text{Count}(w_1^{N-1} w_N)}{\text{Count}(w_1^{N-1})}$$

Example: \rightarrow start of sentence \leftarrow End of sentence

$\langle s \rangle$ I am sam $\langle /s \rangle$ $P(I / \langle s \rangle)$

$\langle s \rangle$ Sam I am $\langle /s \rangle$

$\langle s \rangle$ I do not like Sam $\langle /s \rangle$

Q.1. $P(\text{am} / I)$

Q.2. Prob of sentence $\langle s \rangle$ I am sam $\langle /s \rangle$ (Bigram)

Q.3. Prob of $P(\text{Sam} / \text{do})$ using add-1 smoothing (Laplacian Smoothing)

① $P(\text{am} / I) = \frac{\text{Count}(I \text{ am})}{\text{Count}(I)}$

$$= \frac{2}{3}$$

② Bi-gram Probability

$$P(I / \langle s \rangle) \times P(\text{am} / I) \times P(\text{sam} / \text{am}) \times P(\langle /s \rangle / \text{sam})$$

$$\frac{\text{Count}(\langle s \rangle I)}{\text{Count}(\langle s \rangle)} \times \frac{\text{Count}(I \text{ am})}{\text{Count}(I)} \times \frac{\text{Count}(\text{am sam})}{\text{Count}(\text{am})} \times \frac{\text{Count}(\text{sam} \langle /s \rangle)}{\text{Count}(\text{sam})}$$

$$= \frac{2}{3} \times \frac{2}{3} \times \frac{1}{2} \times \frac{2}{3} = \frac{8}{54}$$

① $P(am/I) = \frac{\text{Count}(I am)}{\text{Count}(I)}$
 $= \frac{2}{3}$

② Bi-gram Probability
 $P(I/\langle s \rangle) \times P(am/I) \times P(sam/am) \times P(\langle s \rangle/sam)$
 $= \frac{\text{Count}(\langle s \rangle I)}{\text{Count}(\langle s \rangle)} \times \frac{\text{Count}(I am)}{\text{Count}(I)} \times \frac{\text{Count}(am sam)}{\text{Count}(am)}$
 $\Rightarrow \frac{2}{3} \times \frac{2}{3} \times \frac{1}{2} \times \frac{1}{3} = \frac{4}{54}$

③ $P(sam/do) = \frac{\text{Count}(do sam) + 1}{\text{Count}(do) + |V|}$
 $= \frac{0+1}{1+6} = 1/7$

laplacian Smoothing

$P(am/I) = \frac{\text{Count}(I am)}{\text{Count}(I)}$
 $= \frac{2}{3}$

Bi-gram Probability
 $P(am/I) \times P(sam/am) \times P(\langle s \rangle/sam)$
 $\times \frac{\text{Count}(I am)}{\text{Count}(I)} \times \frac{\text{Count}(am sam)}{\text{Count}(am)} \times \frac{\text{Count}(sam \langle s \rangle)}{\text{Count}(sam)}$
 $\times \frac{1}{2} \times \frac{2}{3} = \frac{8}{54}$

$P(do) = \frac{\text{Count}(do sam) + 1}{\text{Count}(do) + |V|}$
 $= \frac{0+1}{1+7} = 1/8$

$|V| = \{ I, am, sam, do, not, like, \langle s \rangle \}$

Bi-gram language model

Diagram showing transitions from 'I' to 'am', 'sam', and 'do'. From 'am', transitions to 'like' and 'running'. From 'sam', transition to '⟨s⟩'. Probabilities listed: $P(am/I) = 0.7$, $P(sam/I) = 0.1$, $P(do/I) = 0.2$, $P(sam/am) = 0.6$, $P(like/am) = 0.2$, $P(running/am) = 0.2$.

Perplexity: is a standard metric used to evaluate how well a language model predict a sentence.

Perplexity is inverse probability of test set normalized by the no of words. for a sentence W with N words

$$W = \{w_1, w_2, \dots, w_N\}$$

$$\text{Perplexity} = \left[P(w_1 w_2 \dots w_N) \right]^{\frac{1}{N}} = \sqrt[N]{\frac{1}{P(w_2/w_1) \times P(w_3/w_2) \dots P(w_N/w_{N-1})}}$$

Bigram or Trigram or N-gram
(For Bigram only)

example: \rightarrow

$\langle s \rangle$ I am sam $\langle /s \rangle$ ← End of sentence

$\langle s \rangle$ Sam I am $\langle /s \rangle$

$\langle s \rangle$ I do not like Sam $\langle /s \rangle$

$P(I \text{ like Sam } | S) \Rightarrow$ Bigram

Perplexity Score

$$P(I | S) \times P(\text{like} | I) \times P(\text{Sam} | \text{like}) \times P(S | \text{Sam})$$

(1)

$$P(\text{am} | I) = \frac{\text{Count}(I \text{ am})}{\text{Count}(I)} = \frac{2}{3}$$

Perplexity is inverse probability of test set normalized by the no of words. for a sentence W with N words

$$\sqrt[N]{\frac{1}{P(S)}}$$

$$N = 4$$

Probability

$$P(\text{am} | I) \times P(\text{Sam} | \text{am})$$

$$W = \{w_1, w_2, \dots, w_N\}$$

Perplexity = $\left[P(w_1 w_2 \dots w_N) \right]^{\frac{1}{N}}$
 Bigram or Trigram or N-gram

$$\sqrt[N]{\frac{1}{P(w_2/w_1) \times P(w_3/w_2) \times \dots \times P(w_N/w_{N-1})}} = \frac{1}{3} \times \frac{1}{2} \times \frac{2}{3} = \frac{8}{54}$$

(For Bigram only)