Department of Mathematical Sciences
Rajiv Gandhi Institute of Petroleum Technology, Jais

Tutorial Set - 6

Subject: Statistical Methods & Data Analysis (MA 231) **Instructor: C. Kundu**

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1. Find the maximum likelihood estimate for the parameter λ of a Poisson distribution with probability mass function

$$f(x;\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!}; x = 0,1,2, \dots$$

on the basis of a random sample of size n.

2. Find maximum likelihood estimates of the parameters μ and σ^2 in Normal (μ, σ^2) population for a sample of size n. Show that the MLE of μ is unbiased but the estimate of σ^2 is biased and hence find the unbiased estimator for σ^2.

3. Let $x_1, x_2, \dots, x_n$ be a random sample from the uniform distribution with probability density function :

$$f(x,\theta) = \begin{cases} \frac{1}{\theta}, & 0 < x < \theta \\ 0, & elsewhere \end{cases}$$

Obtain the maximum likelihood estimator for θ.

4. The sponsors of television shows targeted at children wanted to know the amount of time children spend watching television, since the types and number of programs and commercials presented are greatly influenced by this information. As a result, a survey was conducted to estimate the average number of hours Australian children spend watching television per week. From past experience, it is known that the population standard deviation σ is 8.0 hours. The following are the data gathered from a sample of 100 children.

Amount of time spent watching television each week:

39.7	21.5	40.6	15.5	43.9	33.0	21.0	15.8	27.1	23.8	18.3	23.4	20.6	28.4	29.8	41.3	36.8
35.5	27.2	21.0	19.7	22.8	30.0	22.1	30.8	34.7	15.0	23.6	38.9	29.1	28.7	29.3	20.3	36.1
21.6	15.1	43.8	29.0	30.2	26.5	20.5	24.1	29.3	14.7	13.9	37.1	32.5	24.4	22.9	24.5	19.5
29.9	46.4	31.6	20.6	38.0	21.8	23.2	22.0	35.3	17.0	24.4	34.9	24.0	32.9	15.1	23.4	19.5
26.5	42.4	38.6	23.4	37.8	26.5	22.7	27.0	16.4	39.4	38.7	9.5	20.6	21.3	33.5	23.0	35.7
23.4	30.8	27.7	25.2	50.3	31.3	28.9	31.2	15.6	32.8	17.0	11.3	26.9	26.9	21.9		

Find the 95% confidence interval estimate of the average number of hours Australian children spend watching television.

5. A taxi fare is determined by distance travelled as well as the amount of time taken for the trip. In preparing to apply for a rate increase, the general manager of a fleet of taxis wanted to know the distance customers travel by taxi on an average trip. She organized a survey in which she asked taxi drivers to record the number of kilometers (to the nearest one-tenth) travelled by randomly

selected customers. A sample of 41 customers was produced. The results appear below. Distance travelled by taxi (km):

8.2 9.1 11.2 5.0 6.4 9.5 10.1 7.9 8.3 6.8 6.9 7.9 1.1 6.7 11.4 6.9 6.5 8.0 1.5 8.2 7.6
14.1 7.0 10.0 7.1 8.0 8.1 4.4 5.9 2.3 13.3 9.2 2.8 13.0 8.3 10.4 9.0 3.5 9.8 6.5 7.7

Estimate the mean distance travelled by taxi with 95% confidence.

6. A major manufacturer of processed meats monitors the amount of each ingredient. The weight (lb) of cheese per run is measured on n = 80 occasions. (courtesy of David Brauch))

72.2 67.8 78.0 64.4 76.3 72.3 73.1 71.7 66.2 63.3 85.4 67.4 66.3 76.3 57.7 50.3 77.4
73.9 67.4 74.7 68.2 87.4 86.4 69.4 58.0 63.3 72.7 73.6 68.8 63.3 63.3 73.0 64.8
73.1 70.9 85.9 74.4 75.9 72.3 84.3 61.8 79.2 64.3 65.4 66.7 77.2 50.0 70.3 90.4 63.9
62.1 68.2 55.1 52.6 68.5 55.2 73.5 53.7 61.7 47.9 72.3 61.1 71.8 83.1 71.2 58.8 61.8
86.8 64.5 52.3 58.3 65.9 80.2 75.1 59.9 62.3 48.8 64.3 75.4

Construct a 95% confidence interval for the population standard deviation σ.

7. Container-filling machines are used to package a variety of liquids, including milk, soft drinks and paint. Ideally, the resulting amount of liquid in each container should vary only slightly, since large variations will cause some containers to be under filled (cheating the customer) and some to be overfilled (resulting in costly waste). The director of a company that has developed a new type of container-filling machine boasts that it can fill 1-litre (1000 cubic centimeters) containers so consistently that the variance of fills will be less than 1 cubic centimeter. To examine the veracity of the claim, a random sample of 25 1-litre fills was taken and the results (cubic centimeters) recorded. The data are listed below. Sample of 25 '1-litre' fills:

999.6 1000.7 999.3 1000.1 999.5 1000.5 999.7 999.6 999.1 997.8 1001.3 1000.7 999.4
1000.0 998.3 999.5 1000.1 998.3 999.2 999.2 1000.4 1000.1 1000.1 999.6 999.9

Estimate with 98% confidence the variance of fills. Assume that the population of 1-litre fills is normally distributed.