

Time 3 hrs

Full marks 60

Note: Marks for each question are mentioned in square brackets.

Qu 1. (a) What is the role of UI design in frontend development? Name at least two design tools used by frontend developers.

(b) Name and define at least three techniques used for optimizing frontend performance. [2.5]

Qu 2. (a) Create a container that uses 80% of the available width. The container should have a black border and padding. The container must enclose some text, one image and a bulleted list of 2 items. [2.5]

(b) Make an HTML element slide in and out horizontally using CSS animations? [2.5]

(c) What is the use of media queries? [1]

(d) Create the following layout using HTML and CSS:

- The layout consists of a header with the text "Hover Me" inside.
- When you hover over the header, the background color of the header should change to red and the text should be underlined.

(e) Show the output of the following html code: [2.5]

```
<div id="container">
  <p>I failed math so many times at school, I can't even count.</p>
</div>
<span style="color: red;">I used to have a handle on life, but then it
broke.</span>
<p>The problem with <b>kleptomaniacs</b>is that they always
<i>take things</i> literally.</p>
```

(f) What do you understand by cross-browser and cross-device testing? [1]

Qu 3. (a) Create a webpage that displays a button. At every second, the browser window changes its color to a random combination of red, green, and blue (RGB) values. On clicking the button, the color change stops. Use only HTML and Javascript to design this page. Do not use CSS. [4]

(b) Create a simple interactive webpage that displays a heading and a button. Clicking the button should change the background color of the heading to blue. Use only HTML and Javascript to design this page. Do not use CSS.

(c) What is the output of the following code? Explain. [2]

[1]

```

<div id="myParagraph">Never trust atoms; they make up everything.</div>
<script>
let paragraphElement = document.getElementById("myParagraph");
paragraphElement.innerHTML = "<b>Light travels faster than sound, which is
the reason that some people appear bright before you hear them speak.</b>";
</script>

```

(d) What is the difference between prototype and instance members in Javascript?

[1]

Qu 4. (a) Create a React component that displays two buttons, both of which update a shared count value when clicked?

[4]

(b) Create a React component called 'AlertButton' that displays a button and has a specific message which shows up as an alert when the button is clicked. Use this component within another component called Toolbar to create two buttons, one for playing a movie and another for uploading an image.

[3]

Qu 5. (a) What is the difference between `app.post('/', (req, res) and app.put('/user', (req, res)?`

[1]

(b) Write a Node.js code snippet that creates a server listening on localhost (127.0.0.1) port 3000, which responds with "Hello World" when accessed.

[4]

Qu 6. (a) Explain the Model View Controller mechanism.

(b) What will be the output of the following code snippet?

[2]

[2]

```

import json
print(json.dumps({'foo': 1, 'bar': 2}))
x = json.dumps([1, 2, 3, 4])
print("x is a ", type(x))
print(x)
y = json.loads(x)
print("y is a ", type(y))
y.append(5)
print(y)

```

Qu 7. (a) Mention three benefits of NoSQL databases over SQL databases.

[1]

(b) Assume a database of books with the following attributes: Title, Author, Publisher and ASIN. Write a MongoDB query to filter out books written by Ruskin Bond and Paula Hawkins.

[1]

(c) Assuming that you have already connected to a MongoDB database `db` successfully using express. Write down a navigation route that will help you locate a record with a specific id. In case the record is not found, display a proper error message.

[3]

Qu 8. Answer any four:

- (a) Why are effective logging and monitoring mechanisms essential? [1]
- (b) How will the integration of Browser Caching and Content Delivery Network (CDN) improve web performance? [1]
- (c) What is semantic versioning? What are its components? [1]
- (d) What are the types of deployments and what is their specific purpose? [1]
- (e) What is the Success Criterion 2.2.1 regarding **Enough Time**. [1]

Qu 9. In context of your web development project, answer the following:

- (a) System Breakdown: [4]
  - (i) Draw a block diagram representing the major components of your web development project.
  - (ii) Briefly describe the functionalities of each block in the diagram.
  - (iii) Identify the interactions between different blocks.
- (b) Technology Stack: [3]
  - (i) List the key technologies used in your project.
  - (ii) Explain why you chose these specific technologies and how they contributed to achieving your project goals.
- (c) Scalability Considerations: [2]
  - (i) Discuss any potential limitations of the current system architecture in terms of scalability.
  - (ii) Briefly outline one or two strategies you could implement to improve the project's scalability in the future.
- (d) Security Measures: [1]
  - (i) Briefly describe one security measure you implemented in your project to protect user data or prevent unauthorized access.

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# Rajiv Gandhi Institute Of Petroleum Technology, Jais

End Semester Examination (AY 2023 - 24 / Even / Sem - IV)

B.Tech. in CSE, IDD(AI), MC, IT, CSD, PE, CE, REE

NUMERICAL METHODS (MA 221)

Total Marks : 60

Duration : 3 hours

10 - May - 2024

### Instructions.

- i) Answer all the questions.
- ii) Use of **own** calculator is allowed.
- iii) Symbols have their usual meanings.
- iv) For questions having more than one part, answers are to be written together in continuity.

1. Label the following statements (i)-(vi) as **TRUE/FALSE** only, no justification is needed. Each correct answer carries 1 mark. No negative marks for wrong answer. (1 × 6)

- i) For a given data set  $\{(x_i, f(x_i)) : i = 0, 1, \dots, n\}$ , the interpolating polynomials by Lagrange's and Newton's divided-difference methods are the same.
- ii) If  $x_1 \neq x_2$ , then  $f'(x_1)$  and  $f[x_1, x_2]$  are always different.
- iii) Simpson's  $\frac{1}{3}$  rule is exact for polynomials of degree one.
- iv)  $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$  is a positive definite matrix if  $ab > 0$ .
- v) Inverse of a lower triangular matrix (if exists) is a lower triangular matrix.
- vi) Euler's method for solving ODE with initial condition is the Taylor's method of order 1.

2. (a) Define the divided difference of a function for  $n$  points. Find the second order divided difference of  $f(x) = x^2$ , for the points  $x_0, x_1$  and  $x_2$ . (1+2)

(b) Find the error in interpolating  $f(x) = x^{n+1}$  by the polynomial which interpolates at the points  $x_0, x_1, \dots, x_n$ , and then write the interpolating polynomial. (2+1)

(c) Determine the step size  $h$  in a table of equally spaced values of the function  $f(x) = \sqrt{x}$  in the interval  $[1, 2]$ , so that the interpolation with a second degree polynomial will yield an error less than  $5 \times 10^{-8}$ . (5)

(d) If  $f(x) = e^{ax}$ , show that  $\Delta^n f(x) = (e^{ah} - 1)^n e^{ax}$ , where  $h$  is the mesh size. (2)

(e) Let  $g(x) = \ln x$ . Evaluate the first order divided difference  $g[1, 1]$ . Use  $f(0) = 0$ ,  $f(1) = 0.0349$ ,  $f'(0) = 2$ ,  $f'(1) = 1.9988$  to find the cubic interpolating polynomial by divided-difference method and hence evaluate  $f(0.5)$ . (1+4+1)

3. (a) Find the percentage error in the numerical evaluation of the integral  $\int_0^{\frac{\pi}{2}} (8 + 4 \cos x) dx$  with the trapezoidal rule using only the end points and their mid point. (4)

0 0

1 0.0349 1.9988

the trapezoidal rule using only the end points and their mid point.

(b) Let  $f$  be a twice differentiable function on  $[a, b]$ . Establish the error formula in computing  $\int_a^b f(x) dx$  by trapezoidal rule. (3)

(c) Evaluate the integral  $\int_0^2 \frac{2x}{(1+x^4)} dx$ , using Simpson's  $\frac{1}{3}$  rule over the interval  $[0, 1]$ , and the two point Gaussian quadrature rule over the interval  $[1, 2]$ . Compare with the actual value. (4+1)

4. (a) Given the differential equation  $\frac{dy}{dt} = f(t, y)$ , where  $f(t, y)$  depends solely on  $t$ , demonstrate that the Runge-Kutta method of order 4 reduces to Simpson's  $\frac{1}{3}$  rule for numerical integration, resulting in the approximation: (3)

$$\int_{t_n}^{t_{n+1}} f(t) dt \approx \frac{h}{6} \left[ f(t_n) + 4f\left(t_n + \frac{h}{2}\right) + f(t_n + h) \right] = y_{n+1} - y_n.$$

(b) Use the Runge-Kutta formula of fourth order to find the numerical solution at  $x = 0.6$  for

$$\frac{dy}{dx} = \sqrt{(x+y)}, \quad y(0.4) = 0.41.$$

Compare with the Euler's method solution. Assume the step length  $h = 0.2$ . (4+1)

5. (a) Find the inverse of the matrix  $\begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$  using LU decomposition method with  $u_{ii} = 1$ , (4)

for  $i = 1, 2, 3$ .

(b) For the following system of equations (2+3)

$$-3x + y = -2$$

$$2x - 3y + z = 0$$

$$2y - 3z = -1,$$

(i) set up the Gauss-Seidel iteration scheme in matrix form,

(ii) starting with initial guess as  $(0, 0, 0)$ , iterate two times.

6. (a) Let  $f$  be a twice differentiable function on  $[a, b]$ , and let  $\alpha \in [a, b]$  be a zero of  $f$ . If  $x_k$  is the  $k^{\text{th}}$  iteration in the Newton-Raphson method for solving  $f(x) = 0$ , then show that (3)

$$(\alpha - x_{n+1}) = -\frac{1}{2} (\alpha - x_n)^2 \frac{f''(\xi_n)}{f'(x_n)},$$

where  $\xi_n$  is a number between  $\alpha$  and  $x_n$ . [Hint: Use Taylor series expansion with error term.]

(b) Perform one iteration of the Newton-Raphson method to solve the system of equations

$$x^2 + xy + y^2 = 7$$

$$x^3 + y^3 = 9$$

with the initial approximation  $x_0 = 1.5, y_0 = 0.5$ . (3)

— END —

$$\begin{pmatrix} 0 & 9 & 0 \\ 0 & 6 & 9 \\ 0 & 4 & 6 \end{pmatrix}$$

22IT3040

**Rajiv Gandhi Institute of Petroleum Technology, Jais, Amethi**

B.Tech-2<sup>nd</sup> Year (CSE+IDD+IT+MNC+ Minor)/ Even Semester/ End Semester Examination

2023-24

Subject: CS212 Database Management System

Total Marks:60

Time:3 Hours

Instructions:

- i) There are 16 questions in this question paper.
- ii) You have to attempt all the questions.
- iii) Marks against each question are indicated in big brackets.
- iv) Attempt the questions in serial order (if possible).

**Section A**

- Q.1 What is the purpose of the ACID properties in database transactions? Provide a brief explanation of each property. [2]
- Q.2 The record pointer, key field and block pointer of a B tree are 8B, 10B, & 6B respectively. If the block size is 1kB, find the order of the B tree. [2]
- Q.3 What is the highest normal form of the relation R(ABCDEF) having functional dependency set  $F = \{A \rightarrow BC, C \rightarrow AD, E \rightarrow ABC, F \rightarrow CD, CD \rightarrow BEF, AB \rightarrow D\}$  [2]
- Q.4 Describe the concept of data redundancy in the context of database design. How can it be minimized? [2]
- Q.5 What is a deadlock in database management systems? Provide an example scenario and explain how it can be resolved. [2]

**Section B**

- Q.6 What is a 2-phase locking protocol? How does it guarantee serializability. [4]
- Q.7 Find the correct serializable schedule for the following schedule: [4]

<u>T1</u>	<u>T2</u>	<u>T3</u>
Read ( X )		
	Read ( Y )	
		Read ( Y )
	Write ( Y )	
Write ( X )		Write ( X )
	Read ( X )	
	Write ( X )	

Q.8 Does the Timestamp Ordering protocol guarantee freedom from deadlock? Justify your answer with appropriate examples. [4]

Q.9 Let S be the following schedule of operations of three transactions T1, T2, and T3 in a relational database system: R2(Y), R1(X), R3(Z), R1(Y)W1(X), R2(Z), W2(Y), R3(X), W3(Z). Please explain whether schedule S is conflict-serializable. Additionally, if T3 commits before T1 finishes, then is S recoverable? Please explain. [4]

Q.10 Given that a block can hold either 3 records or 10 key pointers, a database contains n records. How many blocks do we need to hold the data file and the dense index? [4]

### Section C

Q.11 Construct a B-tree with the following specifications: [5]

- Order of the B-tree:  $m = 3$  (meaning each node can have at most 3 children)
- Data values to be inserted: [10, 20, 5, 6, 12, 30, 7, 17, 19, 25]

Q.12 Consider a database schema for a library with the following tables: [5]

Books: (book\_id, title, author, category, publication\_year)

Authors: (author\_id, author\_name)

Categories: (category\_id, category\_name)

Users: (user\_id, username, email)

Borrowings: (borrow\_id, user\_id, book\_id, borrow\_date, return\_date)

A) Write an SQL query to find the total number of books borrowed by each user.

select user-id, count(book-id) from users, Borrowings  
 group by user-id  
 having 1

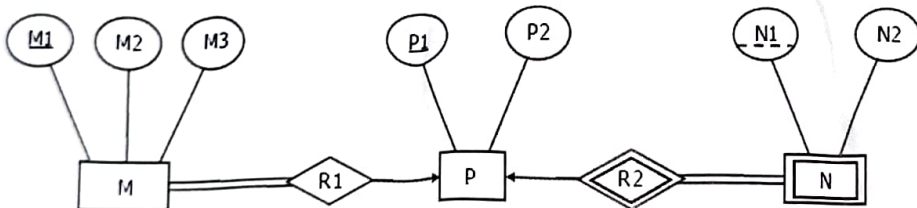
B) Consider a table named Employees with columns: (employee\_id, first\_name, last\_name, salary, department\_id). Write an SQL query to find the average salary of employees in each department, along with the department name.

Q.13 In a B+ tree, if the search-key value is 12 bytes long, the block size is 1024 bytes and the block pointer is 6 bytes, find the maximum number of keys that can be accommodated in each non-leaf node of the tree. [5]

Q.14 Explain the concepts of primary, secondary, and clustered indexing in database management systems (DBMS). Differentiate between them and discuss their advantages and disadvantages. Provide an example scenario for each indexing technique and explain how it affects query performance. [5]

Q.15 Explain the structure and functionality of B and B+ trees in database systems. Discuss their differences, advantages, and use cases. Provide an example scenario for each type of tree and explain how they facilitate efficient data retrieval. [5]

Q.16: Consider the following ER diagram. Find the minimum number of tables needed to represent M, N, P, R1, R2. Specify the attributes for each table. [5]



Q.13  
Q.14  
Q.15  
Q.16

**RAJIV GANDHI INSTITUTE OF PETROLEUM TECHNOLOGY, JAIS AMETHI**  
**B.Tech. 2nd YEAR (CSE, IT, MnC, IDD, EC)**  
**End-SEM EXAMINATION, May 2024**

22173040

**INSTRUCTOR: Dr. NIRBHAY TAGORE**

**SUBJECT: Computer Organization and Architecture (COA)**  
**DATE & TIME: 1<sup>st</sup> May 2024, 11:45 A.M. – 2:45 P.M.**

**COURSE CODE: CS232**  
**FULL MARKS: 60**

**INSTRUCTIONS:**

- I. All the sections are compulsory. Intermediate choices are given in the section itself.
- II. Marks against each question are indicated.
- III. Attempt the questions in serial order (if possible).

**Section-I**

**Q1.** Consider the arrangement given below which uses three 8-bit registers A, B, and C. Working with timing clocks  $T1$ ,  $T2$ ,  $T3$ , and  $T4$  and supports two instructions  $I1$  and  $I2$ . Create hardwired control unit for activating pin Ain, Aout, Cin, and Bout.

	$I1$	$I2$
$T1$	Cin, Aout	Ain, Bout
$T2$	Cout, Bin	Cin, Bout
$T3$	Ain, Bout	Bin, Cout
$T4$	Bin, Cin	Bout, Cin

*Marks: 4*

**Q2.** Calculate the EMAT (Effective M/y Access Time) for a machine with a L1 cache hit ratio of 85%, with cache access time of 5ns. L2 cache hit ratio of 90%, with cache access time of 8ns and main m/y access time is 90ns. for both hierarchical and simultaneous access.

*Marks: 2+2= 4*

**Q3.** Discuss about the cache replacement policies Optimal and LRU. Let the block numbers to be allocated to the cache are 7, 0, 2, 1, 0, 2, 4, 1, 4, 7, 2, 0, 1, 4, 2, 0. Compare the number of cache miss and hit using both optimal and LRU for completing the execution of an instruction requiring these blocks.

*Marks: 2+3 = 5*

**Q4.** Consider the number is in IEEE 754 single precision and bias is 127.

- a.  $S = 0$ ,  $E = 10001000$ ,  $F = 101100000000000000000000$ .
- b.  $S = 1$ ,  $E = 10000001$ ,  $F = 100110000000000000000000$ .
- c.  $S = 0$ ,  $E = 01110001$ ,  $F = 010010000000000000000000$ .

Identify the number in decimal system and highlight all the necessary steps used.

*Marks: 2\*3 = 6*

**Q5.** Draw and explain the flow-chart for non-restoring division algorithm with its advantages over restoring division algorithm. Calculate the remainder and quotient of  $13/2$  by applying no-restoring division algorithm.

*Marks: 2+3 = 5*

Handwritten calculations for Q5 showing binary division of 13 by 2 using the non-restoring division algorithm. The calculations show the step-by-step subtraction and shifting of bits to determine the quotient and remainder.

Q6. Consider 5 different configurations of the system 1 and system 2 using Direct mapping set associative mapping respectively is given below. Complete the table for both the direct and set associative mapping.

Main M/y size	Cache Size	Block Size	No. of Bits in Tag	Tag Directory Size	Associativity (for set associative mapping)
16 GB	64 MB				2-way
32 GB	128 MB				4-way
128 GB	32 MB				4-way
256 GB	256 MB				8-way
1 TB	4 GB				8-way

Marks: 5

Section-II

Marks: 2\*5 = 10

Q7. Describe any five of the following in short:

- a) Hardwired vs Micro-programmed control Unit
- b) Polled vs vectored interrupt
- c) Maskable vs non-Maskable interrupt
- d) DRAM vs SRAM
- e) Compulsory vs conflict vs capacity miss
- f) RISC vs CISC architecture
- g) Indexed vs Relative Addressing Mode

0 00010 00011 0110

Q8. Explain the concept of pipelined architecture in computer organization. Describe how a typical Uni-processing system is different from Multiprocessing and pipelined architecture with diagram.

Marks: 2+3=5

Q9. How asynchronous data transfer is different from synchronous data transfer. Highlight the difference between 2-way and 3-way handshaking mechanism with proper diagram.

Marks: 2+2 = 4

Q10. Elaborately explain the problem of cache coherence. Also provide the possible solution discussed in the class to handle the problem.

Marks: 2+2 = 4

Q11. What do you understand from point-to-point interconnect. Discuss any one example which uses this mechanism along with advantages and disadvantages with proper diagram.

Marks: 2+2 = 4

Q12. How locality of reference improvise system throughput. Explain the different types of localities of reference in detail with appropriate example.

Marks: 2+2 = 4

Handwritten notes for Q12 illustrating locality of reference:

4	M 00010	A 00000 00001 11111	Q 1101 1012 1012	Op 2, 1, 2 SL A = A - M	M = 11101 11110 00010 00000	11110 00010 00000
3	M 00010	A 11111 11111 00000	Q 1010 010? 010?	Op 0 SL A = A + M	Op 0 SL A = A + M	00010 11110 00000
2	M 00010	A 00001 00010 00000	Q 0101 101? 101?	Op 1 SL A = A - M	Op 1 SL A = A - M	11111 00010 00000
1	M 00010	A 00000 00001 00011	Q 1011 011? 011?	Op 0 SL A = A + M	Op 0 SL A = A + M	00001 00010 00011

Rajiv Gandhi Institute of Petroleum Technology, Jais  
B. Tech. (CSE+CSDE+IT+MnC+EE+EV+CE+PE+REE+PP) and IDD  
Even Semester

End - Semester Examination, A.Y. 2023-2024

Subject: MA 231 Statistical Methods and Data Analysis

Time : 3 hours

Full Marks : 60

■ Instructions:

- Answer all the questions. All parts of a question should be answered together.
- Use of calculator is allowed. Statistical Tables will be provided. Do not write anything on the tables. Return the tables after exam.

1. (i) In answering a question on a multiple-choice type test, a student either knows the answer or guesses. Let  $p$  be the probability that the student knows the answer. Assume that a student who guesses at the answer will be correct with probability  $1/m$ , where  $m$  is the number of multiple-choice options. Find the probability that a student answers the question correctly. Also find the conditional probability that a student knew the answer to a question given that he or she answered it correctly.

(ii) A fast food chain puts a winning discount coupon on every fifth package of French fries. Find the probability that one customer will win a coupon in his third or fourth purchase of French fries. [3 + 3]

2. (i) The lifetime in hours of a certain kind of radio tube is a random variable having a probability density function given by

$$f(x) = \begin{cases} 0 & \text{for } x \leq 100 \\ \frac{100}{x^2} & \text{for } x > 100. \end{cases}$$

Assuming independence of the lifetimes of the radio tubes, find the probability that exactly 2 of 5 such tubes in a radio set will have to be replaced within first 150 hours of operation.

(ii) Suppose the lifetime of an SSD hard disk can be modelled using an exponential distribution. It is also believed that the average lifetime of a hard drive is 8 years. Then

(a) Find the probability that an SSD hard disk lasts more than 7 years.

(b) Find the probability that a 3 year old SSD drive will last 7 years more. [4 + 3]

3. Suppose the length of human pregnancies from conception to birth follows a normal distribution with a mean of 266 days and a standard deviation of 50 days. Find the probability that the average length of pregnancy will last between 250 and 270 days for a selected sample of 25 pregnant women. [3]

4. Find the maximum likelihood estimator (MLE) of the the mean  $\mu$  of an exponential population, based on an iid random sample of size  $n$ . Show that the MLE of  $\mu$  is unbiased and consistent. [8]

5. (i) A researcher wants to measure the income level of employees working in IGL. The total employee strength of IGL is 1200. A random sample of 50 employees reveals that the average income of sampled employees is Rs. 15,000. Historical data reveals that the S.D. of the income of the employees is approx. Rs. 1500. Construct a 99% C.I. for obtaining the average income of all the employees working here. [P.T.O.]

(ii) The service time in queues should not have a large variance; otherwise, the queue tends to build up. The ICICI bank regularly checks service time by its tellers to determine its variance. A random sample of 22 service times (in minutes) gives  $s^2 = 8$ . Give a 95% confidence interval for the variance of service time at the bank. [3 + 4]

6. (a) A business student claims that on an average an MBA student is required to prepare more than five cases per week. To examine the claim, a statistics professor asks a random sample of 10 MBA students to report the number of cases they prepare weekly. The results are exhibited here.

2 7 4 8 9 5 11 3 7 4

Can the professor conclude at 5% significance level that the claim is true, assuming that the number of cases is normally distributed?

(b) A telescope manufacturer wants its telescopes to have standard deviations in resolution to be significantly below 2 when focusing on objects 500 light-years away. When a new telescope is used to focus on an object 500 light-years away 30 times, the sample standard deviation turns out to be 1.46. Should this telescope be sold? Test your hypotheses at 1% sig. level. [4 + 3]

7. (i) Mr. A. Srivastava, the current manager of RGIPT Jais campus branch, BoB is interested in the percentage of customers who are totally satisfied with the service of the branch. The previous manager had 89% of the customers totally satisfied, and Mr. Srivastava claims that the percentage of totally satisfied customers has been increased in his tenure. He sampled 187 customers and found 157 were totally satisfied. At the 1% sig. level, is there evidence that current manager's claim is valid?

(ii) An automobile manufacturing company wants to launch a new fuel-efficient car. For conducting pre-production research, the company has taken random samples from two cities: Nagpur and Nasik. The average amount spent on purchasing fuel by 8 families in Nagpur is Rs. 5000/- with a variance of Rs. 560/- and by 10 families of Nasik is Rs. 2900/- with a variance of Rs. 997/-. Determine whether there is a significance difference in the variance of the amount spent on the purchase of fuel by families in two different cities. Use 0.02 sig. level. [3 + 4]

8. A large city fire department calculates that for any given precinct, during any given 8-hour shift, there is a 30% chance of receiving at least one fire alarm. Here is a random sampling of 60 days:

No. of shifts during which alarms were received:	0	1	2	3
No. of days:	16	27	11	6

At the 0.05 level of sig., do these fire alarms follow a binomial distribution? [8]

9. A volunteer group, provides from one to nine hours each week with disabled senior citizens. The program recruits among community college students, four-year college students, and nonstudents. A sample of the adult volunteers and the number of hours they volunteered per week is given.

Type of Volunteer	1-3 Hours	4-6 Hours	7-9 Hours	Row Total
Community College Students	111	36	48	255
Four-Year College Students	96	133	61	290
Nonstudents	91	150	53	294
Column Total	298	379	162	839

Is the number of hours volunteered independent of the type of volunteer? Test at a 5% sig. level. [7]

END

Time: 3 hrs

Full Marks: 50

Note: Attempt all questions. The marks of the questions are provided along with the questions.

Every algorithm/ solution must be supported by its Time and Space complexity analysis.

1. Solve the following recurrence relation. Show all the necessary steps.

Q 1(a)	Q 1(b)	Q 1(c)
$T(n) = \begin{cases} 1 & \text{for } n = 2 \\ T(\sqrt{n}) + 2 & \text{for } n > 2 \end{cases}$	$T(n) = 2T(n-1) + 1$	$T(n) = T(n/2) + T(n/4) + \log(n)$

[6]

2. Suppose you have given two strings:  $S_1 = [A, B, C, D]$  and  $S_2 = [A, C, B, A, D]$ , now

- Find the length of the LCS between  $S_1$  and  $S_2$  using a Dynamic Programming approach. Provide the DP formulation and the DP table. Analyze its time and space complexity.
- Suppose you are allowed to insert exactly one character into  $S_1$  to maximize the length of the LCS with  $S_2$ . Determine which character to insert and its position in  $S_1$ . How does it affect the DP table? Justify.
- Explain how the Karatsuba algorithm improves the time complexity of the multiplication of two numbers.

[6]

3. Assume for a quiz, I want all students to be seated in the auditorium according to the ascending order of their heights. Being lazy, you all arrive at different times, one by one, in a random time interval.

- Design an efficient sorting algorithm that ensures that at any point in time, the students who have already arrived are seated in ascending order. (Hint: try to do better than  $O(n^2)$  time complexity)
- What if all the students arrive at once? Can you propose a better algorithm for this case?

[5]

4. Prove that the Clique Problem is NP-complete by reducing it from the SAT Problem. Use the following SAT formula as an example:

$$\text{SAT Formula: } (x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_3) \wedge (x_2 \vee \neg x_3 \vee x_4)$$

[5]

- What is the key difference between a heuristic and an approximation algorithm? Highlight problems where Simulated Annealing can be applied efficiently.
- Explain how the concept of "relaxation" is used to get the 2-approximation algorithm for vertex cover.

[6]

5. (a) Prove that the fractional knapsack can be solved using the greedy method, but the 0-1 knapsack cannot.

(b) Can you develop a greedy approximation algorithm for 0-1 knapsack? Establish its approximation ratio proof.

[6]

7. Let's assume you have a directed graph as shown in figure 1, with a source node marked as 0 and a sink node marked as 5, and each edge has a capacity that represents the maximum amount of flow that can be sent through it.

Your task is to develop an algorithm to find the maximum amount of flow that can be sent from source 0 to sink 5, while respecting the capacity constraints on the edges.

[5]

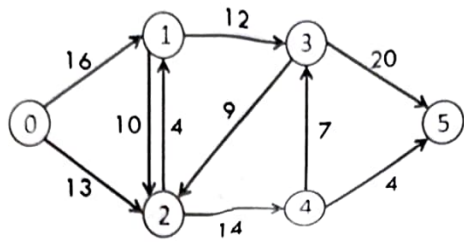


Figure 1: Flow diagram with max edge capacity

8. Your friends have a habit of borrowing money from one another. They return money using bank transactions, which charge a fixed amount of processing fee per transaction. They want to minimize this fee. An example of this is given in figure 2 with the borrowing status of three friends named  $P_0$ ,  $P_1$ , and  $P_2$ . Here the first (left) figure shows the original transaction status, and the second (right) figure shows the optimized transaction status.

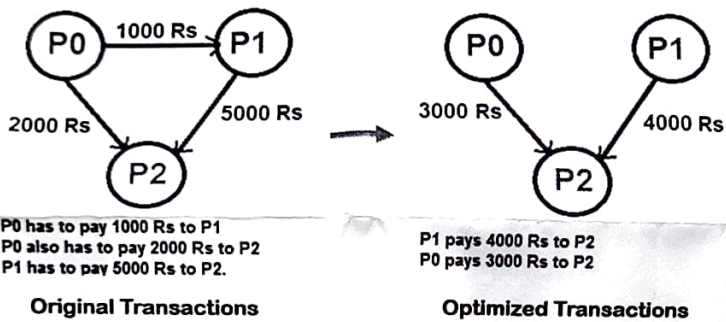


Figure 2: Transaction state: before and after optimization

- (a) Your task is to design an efficient algorithm to minimize the transactions to pay the minimum transaction charges. Analyze its time and space complexity too.
- (b) Dry run your algorithm over the graph given in figure 1. Assume the vertices are friends and edge weights as cash flow.

[5]

9. For the graph given in the figure 1, answer the following:

- (a) Generates a Minimum Spanning Tree using *Prims* Algorithm. What will be the time complexity?
- (b) will I get the 2nd-best MST by deleting the smallest edge from the graph and then running the *Prims* algorithm? Justify your answer. Can you suggest a better approach for this?
- (c) If all the edges in a graph have unique weights then there will be only one MST. True or False? Justify your answer.

[6]