Assignment 12:Numerical analysis
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1. Solve the IVP $y' = 2t - y$, $y(0) = -1$ with $N = 10$, to get the values of y at $t = 1$, using Euler method, also find the truncation error of Euler method?
2. Use the Euler method to solve IVP
$$y' = -2ty^2, y(0) = 1$$
with $h = 0.2, 0.1$ and 0.05 on the interval $[0, 1]$. Determine the bound for the Error.
3. Solve the following ODE using Modified Euler method:
   (a) $y' = 3(t-1)^2$ with $y(0) = 1, h = 0.1$, calculate $y_1\ y_2, \cdots\ y_{10}$.
   (b) $y' = ye^t$ with $y(0) = 1, h = 0.1$, calculate $y_1\ y_2, \cdots\ y_5$.
4. Use Range-Kutta of order 2 and 4 methods to solve the IVP $y' = 0.5(x-y)$ for all $x \in [0, 3]$ with initial condition $y(0) = 1$. Compare the solutions for $h = 1, 0.5, 0.25, 0.125$ along with the exact solution $y(x) = 3e^{\frac{-x}{2}} + x - 2$
5. Consider the initial value problem $y' = xy, y(0) = 1$. Estimate the error at $x = 1$ when Euler method is used to find the approximate solution to this problem with step size $h = 0.01$. (Answer: $|en| \leq 0.03$).
6. Use RK-2 and RK-4 to approximate the solution of following IVPs in the interval indicated:
   (a) $y' = \cos(ty)$,$[0, 1.2]$ with $y(0) = 2$ and $N = 10$.
   (b) $y' = -ye^t$,$[0, 1]$ with $y(0) = 3$ and $N = 10$.
   (c) Given IVP $y' = -2ty^2$ with $y(0) = 1$ and $h = 0.2$ on the interval $[0, 0.4]$. Use Rk-2 and Rk-4 and compare with exact solution $y = \frac{1}{(1+t^2)}$.
7. When $f(t, y)$ depends only on t, show that the RK-4 reduces to Simpson's rule:
$$\int_{t_n}^{t_{n+1}} f(t)dt \approx \frac{h}{6}[f(t_n) + 4f(t_n + \frac{h}{2}) + f(t_n + h)] = y_{n+1} - y_n.$$