Subject: **Numerical Methods** (MA221), problem set 11
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1. Consider the quadrature rule
$$\int_a^b f(x)dx = \sum_{i=0}^{n} w_i f(x_i)$$
where $w_i > 0$ and the rule is exact for $f(x) = 1$. If $f(x_i)$ are in error almost by 0.5×10^{-k}, show that the error in the quadrature rule is not greater than $0.5 \times 10^{-k}(b-a)/2$.

2. There is three-point quadrature formula of the form
$$I_2 = w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3)$$
where $-1 \leq x_1 \leq x_2 \leq x_3 \leq 1$ and $w_1, w_2, w_3 > 0$ to calculate the integral $\int_{-1}^{1} f(x)dx$. Find w_1, w_2, w_3, x_1, x_2 and x_3 so that $I_2 = \int_{-1}^{1} f(x)dx$ for $f(x) = 1, x, x^2, x^3, x^4$ and x^5.

3. Determine the coefficients w_0, $w_1$and w_2 such that the formula
$$\int_0^{2h} \frac{1}{\sqrt{x}} f(x)dx = \sqrt{2h}(w_0 f(0) + w_1 f(h) + w_2 f(2h)),$$
is exact for polynomials of as high order as possible.

4. Use the one-point, two-point and three point Gaussian quadrature rule to approximate
$$\int_{-1}^{1} \frac{dx}{x+2} \ (\approx 1.09861)$$
and compare the result with the trapezoidal rule and Simpson's rule.

5. Use the one point, two-point and three point Gaussian quadrature rule to approximate the integral
$$\int_0^3 x^2 e^x dx.$$