Subject: **Numerical Methods** (MA221) Problem Set 9
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1. Let $f(x) = \log(1 + x)$, $x_0 = 1$ and $x_1 = 1.1$. Use linear interpolation to calculate an approximate value for $f(1.04)$, and obtain a bound on the truncation error.

2. Determine the maximum step size that can be used in the tabulation of $f(x) = \sqrt{1 - x}$ in [0, 1], so that the error in the linear interpolation will be less than 5×10^{-4}. Find also the step size if quadratic interpolation is used.

3. Use Lagrange's formula to express each of the following functions as a sum of partial fractions.

   a) $f(x) = \frac{3x^2+x+1}{(x-1)(x-2)(x-3)}$ b) $f(x) = \frac{x^2+6x-1}{(x^2-1)(x-4)(x-6)}$ c) $f(x) = \frac{x^2+x-3}{x^3-2x^2-x-2}$

4. Find the missing term in the following table using
   (a) Lagrange's interpolation
   (b) divided difference interpolation

x	0	1	2	3	4
y	1	3	9	-	81

5. A curve passes through the points (0, 18), (1, 10), (3, −18) and (6, 90). Find the slope of the curve at $x = 2$.

6. Consider the following table. Using (i) Newton's divided difference interpolating polynomial, (ii) Lagarange's interpolating polynomial, evaluate $f(8)$ and $f(15)$.

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

7. Find the bound for the error in linear interpolation.

8. For linear interpolation, in the case of equispaced tabular data, show that the error does not exceed 1/8 of the second difference.

9. Given the square of the integers N and $N + 1$, what is the largest error that occurs if linear interpolation is used to approximate $f(x) = x^2$ for $N \leq x \leq N + 1$?

10. Determine the step size h in a table of equally spaced values of the function $f(x) = \sqrt{x}$ in the interval $[1, 2]$, so that interpolation with a second degree polynomial will yield a desired accuracy.

11. How much larger could we make step size h so that quadratic interpolation would have an error comparable to that of linear interpolation of $\log_{10} x$ in the interval $[1, 5]$ with $h = 0.01$?

12. Determine the step size that can be used in the tabulation of $f(x) = \sin(x)$ in the interval $[0, \pi/4]$ at equally spaced nodal points so that the truncation error of the quadratic interpolation is less than 5×10^{-8}.

13. A table of values for $f(x) = e^{3x}$ in $[0, 1]$ is constructed with step size 0.05. Find the maximum total error if quadratic interpolation is to be used to interpolate in this interval.

14. Determine the step size that can be used in the tabulation of a function $f(x)$, $a \leq x \leq b$, at equally spaced nodal points so that the truncation error of the quadratic interpolation is less than ϵ.

15. Determine the step size that can be used in the tabulation of a function $f(x)$, $a \leq x \leq b$, at equally spaced nodal points so that the truncation error of the cubic interpolation is less than ϵ.

16. Denoting the interpolant of $f(x)$ on the set of (distinct) points $x_0, x_1, \ldots, x_n$ by $\sum_{k=0}^{n} l_k(x) f(x_k)$, find an expression for $\sum_{k=0}^{n} l_k(0) x_k^{n+1}$.

17. Let $x_0, x_1, \ldots, x_n$ be $n+1$ distinct nodes in the closed interval $[a, b]$ and let $f(x)$ be $n+1$ times continuously differentiable function on $[a, b]$. Then show that

(a) $\frac{d}{dx} f[x_0, \ldots, x_{i-1}, x] = f[x_0, \ldots, x_{i-1}, x, x]$.

(b) the divided difference are symmetric function of their arguments, i.e., for an arbitrary permutation π of the indices $0, 1, \ldots, i$ we have $f[x_0, \ldots, x_i] = f[x_{\pi 0}, \ldots, x_{\pi i}]$.

18. Let $f(x)$ be a real-valued function defined on $I = [a, b]$ and k times differentiable in (a, b). If $x_0, x_1, \ldots, x_k$ are $k+1$ distinct points in $[a, b]$, then show that there exist a $\xi \in (a, b)$ such that

$$f[x_0, \ldots, x_k] = \frac{f^{(k)}(\xi)}{k!}.$$

Assignment Problems:

1. Using Lagrange interpolation, find the unique polynomial $P(x)$ of degree 2 or less such that $P(1) = 1, P(3) = 27, P(4) = 64$.

2. Using Newton's forward interpolation formula show that

$$\sum n^3 = \left\{ \frac{n(n+1)}{2} \right\}^2.$$

3. From the following table, estimate the number of students who obtained marks between 40 and 45:

Marks	30-40	40-50	50-60	60-70	70-80
No. of students:	31	42	51	35	31

4. Find the distance moved by a particle and its acceleration at the end of 4 seconds, if the time verses velocity data is as follows:

t:	0	1	3	4
v:	21	15	12	10