Subject: **Numerical Methods** (MA221) Problem Set 8
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1. Recall the definitions of the following finite difference operators:

   - The shift operator: $E\ f(x_i) = f(x_i + h)$.
   - The forward difference operator: $\Delta\ f(x_i) = f(x_i + h) - f(x_i)$.
   - The backward difference operator: $\nabla\ f(x_i) = f(x_i) - f(x_i - h)$.
   - The central difference operator: $\delta\ f(x_i) = f(x_i + \frac{h}{2}) - f(x_i - \frac{h}{2})$.
   - The average operator: $\mu\ f(x_i) = \frac{1}{2}\left[f(x_i + \frac{h}{2}) + f(x_i - \frac{h}{2})\right]$.

   Show the following.

   a) $\delta = \nabla(1-\nabla)^{-1/2}$

   b) $\mu = \left[1 + \frac{\delta^2}{4}\right]^{1/2}$

   c) $\Delta - \nabla = \Delta\nabla$

   d) $\Delta + \nabla = \Delta/\nabla - \nabla/\Delta$

   e) $\sum_{k=0}^{n-1} \Delta^2 f_k = \Delta f_n - \Delta f_0$

   f) $\mu = \left(1 + \frac{1}{2}\Delta\right)(1+\Delta)^{-1/2} = \left(1 - \frac{1}{2}\nabla\right)(1-\nabla)^{-1/2}$

2. Do the finite difference operators commute with each other? For example, is it true that $\Delta\,\nabla = \nabla\,\Delta$, and similarly for other pairs of operators?

3. Take $f(x)$ to be a polynomial of degree three of your choice, and choose at least five distinct nodal points (with equal spacing for the first three tables, and non-equispaced points for the d.d. table). Then construct the forward, backward, central and divided difference tables for this choice of your function and nodes. Write down your observations about these tables.

4. Calculate the n^{th} divided difference of $f(x) = \frac{1}{x}$.

5. The following data are taken from a polynomial of degree ≤ 5. What is the degree of the polynomial?

x	-1	0	1	2	3
$f(x)$	1	1	1	7	25

6. Let $x_0 < x_1$, and $f(x)$ be defined on the interval $[a, b]$ containing the points x_0, x_1. Assume that $P(x) = c + mx$ satisfies $P(x_0) = f(x_0)$ and $P(x_1) = f(x_1)$. Find the values of c and m. Show that $P(x)$ can be written as $P(x) = f(x_0) + (x - x_0)f[x_0, x_1]$.

7. Let $\cdots < x_{-2} < x_{-1} < x_0 < x_1 < x_2 < \cdots$ be equally spaced nodes with spacing h. Using the principle of mathematical induction, for any fixed k and any m prove the following.

   a) $f[x_k, x_{k+1}, \ldots, x_{k+m}] = \frac{\Delta^m f(x_k)}{m!\,h^m}$

   b) $f[x_k, x_{k-1}, \ldots, x_{k-m}] = \frac{\nabla^m f(x_k)}{m!\,h^m}$

8. Find a relation between

   a) $\Delta^2 f(x)$ and $f''(x)$

   b) $\nabla f(x)$ and $f'(x)$

   c) $\delta^2 f(x)$ and $f''(x)$.

   In each of the above cases, find the order of error.

9. Complete the following data set using Newton's forward/backward difference table. Assume that the data is of some polynomial function.

x	2	3	4	5	6	7	8	9	10
y	?	4.8	8.4	14.5	23.6	36.2	52.8	73.9	?

10. Consider the matrix

$$V = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{bmatrix}.$$

Prove that

$$\det(V) = \prod_{1 \le i < j \le n} (x_j - x_i).$$