

1. Solve the following systems using Jacobi method upto three iterations with initial guess zero:-

$$(a) \begin{aligned} 5x - 2y + 3z &= -1 \\ -3x + 9y + z &= 2 \\ 2x - y - 7z &= 3. \end{aligned}$$

$$(b) \begin{aligned} 2x - y &= 7 \\ -x + 2y - z &= 1 \\ -y - 2z &= 1. \end{aligned}$$

$$(c) \begin{aligned} 7x - 2y + z &= 17 \\ x - 9y + 3z - w &= 13 \\ 2x + 10z + w &= 15 \\ x - y + z + 6w &= 10 \end{aligned}$$

2. Solve the following system using Gauss-Seidel method upto three iterations with initial guess zero-

$$(a) \begin{aligned} 7x - 2y + z &= 17 \\ x - 9y + 3z - w &= 13 \\ 2x + 10z + w &= 15 \\ x - y + z + 6w &= 10 \end{aligned}$$

$$(b) \begin{aligned} 4x + y + z &= 2 \\ x + 5y + 2z &= -6 \\ x + 2y + 3z &= -4. \end{aligned}$$

3. Solve the above system in question 2 by SOR method with $\omega = 1.12$. upto three iterations with initial guess zero.

4. Determine the rate of convergence of the Jacobi method and Gauss-Seidel method for the system: -

$$(a) \begin{aligned} 4x + 0y + 2z &= 4 \\ 0x + 5y + 2z &= -3 \\ 5x + 4y + 10z &= 2. \end{aligned}$$

$$(b) \begin{aligned} 3x + y + z &= 4 \\ -2x + 4y + 0z &= 1 \\ -x + 2y - 6z &= 2. \end{aligned}$$

5. Given the matrix $A = I + L + U$ where $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$, L and U are strictly lower and upper triangular matrix respectively, decide whether Jacobi and Gauss Seidel method converge to $Ax = b$.

6. Given the matrix $A = D + L + U$ where $A = \begin{bmatrix} 3 & 1 & 1 \\ -2 & 4 & 0 \\ -1 & 2 & 6 \end{bmatrix}$, L and U are strictly lower and upper triangular matrix respectively, decide whether Jacobi and Gauss Seidel method converge to $Ax = b$.

7. The system of equations $Ax = b$ is to be solved iteratively by

$$x_{n+1} = Hx_n + b$$

suppose $A = \begin{bmatrix} 1 & k \\ 2k & 1 \end{bmatrix}$, $k \neq \frac{\sqrt{2}}{2}$, k real

Find a necessary and sufficient condition on k for convergence of Jacobi method.

8. Given the system $\begin{aligned} 3x + y + z &= 4 \\ -2x + 4y + 0z &= 1 \\ -x + 2y - 6z &= 2. \end{aligned}$ this system can be solved by the relaxation method. Write down the iteration formula exactly.

9. Solve the system of equations

$$4x + y + z = 2$$

$$x + 5y + 2z = -6$$

$$x + 2y + 3z = -4.$$

using Jacobi iteration method and find it in error format. Take initial guess as $x^0 = [0.5, -0.5 - 0.5]^T$ and perform three iterations in each case. Same as do for Gauss Seidel method.