

1. Solve the following systems using Gaussian Elimination method :-

$$(a) \begin{aligned} x + y &= 3 \\ 3x - 2y &= 4 \end{aligned}$$

$$2x - 2y = -6$$

$$(b) \begin{aligned} x - y + z &= 1 \\ 3y - 2z &= -5 \end{aligned}$$

$$x + 2y + z = 0$$

$$(c) \begin{aligned} 2x + 2y + 3z &= 3 \\ -x - 3y &= 2 \end{aligned}$$

2. Solve the following system using Gaussian Elimination method -

$$x + y + z + w = 10$$

$$2x + 3y + z + 5w = 31$$

$$-x + y - 5z + 3w = -2$$

$$3x + y + 7z - 2w = 18.$$

3. Let $A = (a_{ij})_{n \times n}$ where $a_{ij} = \frac{1}{i+j-1}$, $i, j = 1, 2, \dots, n$

(a) Compute the matrix A for $n = 5$.

(b) $b = [1, 0, 0, 0, 0]^T$, solve the system $Ax = b$ using Gaussian Elimination.

4. Solve the following systems using Gaussian Elimination method: -

$$2x + y + z = 7$$

$$(a) \begin{aligned} 2x + 2y + 3z &= 10 \\ -4x + 4y + 5z &= 14 \end{aligned}$$

$$-4x + 4y + 5z = 14$$

$$x - y + 5w = 18$$

$$(b) \begin{aligned} 3x - 2y + z - w &= 8 \\ x + y + 9z + 4w &= 47 \end{aligned}$$

$$x - 7y + 2z + 3w = 31.$$

5. Solve the following systems using LU decomposition, when $u_{11} = u_{22} = u_{33} = 1$ in upper diagonal matrix: (Crout's method)

$$(a) \begin{aligned} 4x + y + z &= 4 \\ x + 4y - 2z &= 4 \\ 3x + 2y - 4z &= 6 \end{aligned}$$

$$3x + 2y - 4z = 6$$

$$x + y - z = 2$$

$$(b) \begin{aligned} 2x + 3y + 5z &= -3 \\ 3x + 2y - 3z &= 6. \end{aligned}$$

$$3x + 2y - 3z = 6.$$

6. Solve the above systems using LU decomposition, when $L_{11} = L_{22} = L_{33} = 1$ in lower diagonal matrix (Doolittle's method).

7. Can you write the matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ in the form of LU decomposition?

8. Can you write the matrix $A = \begin{bmatrix} 1 & 2 & 6 \\ 4 & 8 & -1 \\ -2 & 3 & 5 \end{bmatrix}$ in the form of LU decomposition?

9. Write in the form of LU decomposition of the following matrices :-

$$(a) A = \begin{bmatrix} 16 & 4 & 4 \\ 4 & 26 & 6 \\ 4 & 6 & 11 \end{bmatrix}$$

$$(b) B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ -2 & 3 & -2 \end{bmatrix}$$