Subject: **Numerical Methods** (MA221), problem set 10
Course Instructor(s): PD+SK+GR
Department of Mathematical Sciences
Rajiv Gandhi Institute of Petroleum Technology, Jais, U.P. 229304

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1. With $f(x) = \ln(x)$, calculate $f(1.5)$ by cubic interpolation, using $f(1) = 0$, $f(2) = 0.693147$, $f'(1) = 1$, $f'(2) = 0.5$.

2. Given the following values of $f(x)$ and $f'(x)$

x	$f(x)$	$f'(x)$
-1	1	-5
0	1	1
1	3	7

estimate the values of $f(-0.5)$ and $f(0.5)$ using the Hermite interpolation. The exact values are $f(-0.5) = 33/64$ and $f(0.5) = 97/64$.

3. Construct the Hermite interpolating polynomial that fits the data

x	$f(x)$	$f'(x)$
1	7.389	14.778
2	54.598	109.196

Estimate the value of $f(1.5)$. Find a bound on the error. If the data is representing the function $f(x) = e^{2x}$, find the actual error at $x = 1.5$.

4. Apply the Rectangle, Trapezoidal and Simpson's methods to evaluate

   a) $\int_0^1 e^{-x^2} dx$
   b) $\int_0^\pi \sin^3(x)\cos^4(x)dx$
   c) $\int_0^\pi \frac{dx}{5+4\cos(x)}$
   d) $\int_0^1 (1 + e^{-x}\sin(4x))dx$.

5. Evaluate the integral of the following tabular data with (i) the trapezoidal rule (ii) Simpson's rule:

x	0	0.1	0.2	0.3	0.4	0.5
$f(x)$	1	8	4	3.5	5	1

6. Find the value of the integral $I = \int_0^1 \frac{dx}{1+x}$ using (i) trapezoidal rule (ii) Simpson's rule. Obtain a bound for the errors. The exact value of I is $\ln 2 \approx 0.693147$.

7. Write down the errors in the approximation of $\int_0^1 x^4 dx$ and $\int_0^1 x^5 dx$ by the Trapezoidal rule and Simpson's rule. Hence find the value of the constant C for which the Trapezoidal rule gives the exact result for the calculation of $\int_0^1 (x^5 - Cx^4)dx$.

8. (a) In $\int_1^3 e^{x^2} dx$ with 10 subintervals, how far off can the (i) trapezoidal rule (ii)Simpson's rule be?
   (b) How large should we chose the number of subintervals so that (i) trapezoidal rule (ii)Simpson's rule approximations to the above integral is certainly within 0.5 of the right value?

9. Evaluate $\int_0^2 e^x dx$ using Simpson's rule with $h = 1$ and $1/2$. Find bound on the error in each case. Compare with the exact solution.

10. Find the number of subintervals and the step size h so that the error for the composite trapezoidal rule is less than 5×10^{-9} for the approximation $\int_2^7 \frac{dx}{x}$.