Assignment 7:Newton method
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1. Performance four iterations of the Newton method to obtain the root of the equation $f(x) = 0$ where $f(x)$:
   (a) $f(x) = x^3 - x^2 - 1$, $x_0 = 1$,
   (b) $f(x) = 3x^3 - x + 4$,$x_0 = -1$,
   (c) $f(x) = x^3 - x^2 + e^x$,$x_0 = -1$,
   (d) $f(x) = e^x + x\sin(x)$, $x_0 = 0$.
2. Calculate an approximate value for $\sqrt{5}$ using Newton's method with $x_0 = 5$.
3. Use Newton's method to compute a zero of the function
$$f(x) = x^5 - 3x^3 - 5x + 4.$$
   to an accuracy of 10^{-6}. Use $x_0 = 1$.
4. The equation $x^2 + ax + b = 0$ has two real roots,α and β. Show that the iteration method
$$x_{k+1} = -\frac{(ax_k + b)}{x_k}$$
   is convergent near $x = \alpha$ if $|\alpha| > |\beta|$ and that
$$x_{k+1} = -\frac{b}{x_k + a}$$
   is convergent near $x = \alpha$ if $|\alpha| < |\beta|$.
5. Consider the function $f = x^3 - 2x + 2$. Let $x_0 = 0$. Show that the sequence $x_1, x_2, \cdots$ fails to approach a root of f, using Newton method.
6. Show that the following sequence have convergence of the second order with the same limit $\sqrt{a}$,
$$x_{k+1} = \frac{1}{2}x_k\left(1 + \frac{a}{x_k^2}\right)$$
   .
7. Solve the nonlinear system
$$f_1(x,y) = x^3 + 3y^2 - 21 = 0, \quad f_2(x,y) = x^2 + 3y + 2 = 0$$
   by Newton's method starting, with the initial estimate $x(0) = (1, -1)$.
8. Find the Jacobian matrix $J(x,y)$ at the point $(-1, 4)$ for the functions
$$f_1(x,y) = x^3 - y^2 + y = 0, \quad f_2(x,y) = xy + x^2 = 0$$
9. Solve the nonlinear system
$$f_1(x,y) = 3x - y - 3 = 0, \quad f_2(x,y) = x - y + 2 = 0$$
   by Newton's method, with the initial estimate $x(0) = (0, 0)$.