Subject : **Numerical Methods** (MA221)
Problem Set 5/Tut: 6 (14.02.25)(Numerical Solutions of non-linear equations)
Course Instructor(s): SK+GR+PD
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1. Find an interval in which the function $f(x) = x^3 - x^2 - 1$ has only one real root.

2. Performance five iterations of the bisection method to obtain the smallest positive root of the equation $f(x) = 0$ where $f(x)$:
   (a) $f(x) = x^3 - 5x + 1$,$x \in (0, 1)$,
   (b) $f(x) = \cos(x) - xe^x$,$x \in (0, 1)$,
   (c) $f(x) = e^{2x} - 7x$, $x \in (0, 1)$.

3. Verify that the function $f(x) = x^2 \sin(x) + 2x - 3$ has exactly one root in $(0, 2)$. Find this root by using the bisection method with an error of no more than 10^{-5} (error tolerance).

4. Consider the function
$$f(x) = x^4 - 5x^3 + \frac{22}{3}x^2 - \frac{116}{27} + \frac{8}{9}.$$
   (a) Check that $f(x)$ has a root α_1 between 0 and 1 and another root α_2 between 1 and 4.
   (b) Compute both roots using the bisection method.

5. Consider the function
$$f(x) = (x - 2)^2 - \ln(x)$$
   on the interval $[1, 2]$.
   (a) Prove that there is exactly one root of this equation in this interval.
   (b) Use the bisection method to approximate a root to 5 digits accuracy.
   (c) How many iterates of the Bisection method are needed to find an approximation to the root of $f(x) = 0$ in the interval to within an accuracy of 10^{-4}?

6. Performance four iterations of the Regula-Falsi method to obtain the root of the equation $f(x) = 0$ where $f(x)$:
   (a) $f(x) = x^3 - 5x + 1$, $x \in (0, 1)$,
   (b) $f(x) = x^3 - x^2 - 1$, $x \in [1, 2]$,
   (c) $f(x) = \cos(x) - xe^x$,$x \in (0, 1)$,
   (d) $f(x) = e^{2x} - 7x$, $x \in (0, 1)$.

7. Given the equation $f(x) = 0$, obtain an iterative method (Regula-Falsi) using the rational approximation
$$f(x) = \frac{x - a_0}{b_0 + b_1 x}$$
   where the coefficients a_0, b_0 and b_1 are determined by evaluating $f(x)$ at x_k, x_{k-1} and x_{k-2}. Carry out two iterations using this method for the equation $2x^3 - 3x^2 + 2x - 3 = 0$ with $x_0 = 0$, $x_1 = 1$, and $x_2 = 2$.

8. Given the equation $f(x) = x^3 - 4x^2 + 2x - 8 = 0$ with accuracy 10^{-4} by using the Regula -Falsi method with $x_0 = 0$ and $x_1 = 1$.