

Department of Mathematical Sciences
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NUMERICAL METHODS (MA 221)

07.02.25 / February 2025

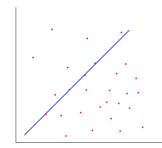
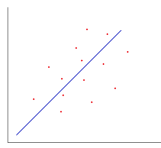
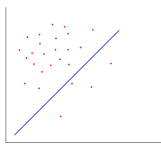
Problem Set 4/Tutorial 5

GR,SK, PD

Least-square method for curve fitting

■ **Tutorial Problems.**

1. Which one of the following graphs best fits the given data points:



Justify your answer.

2. True/False with justification:

- (a) The least squares method is reliable for all data.
- (b) The least squares method gives the predictive model to describe the relationship between two variables.
- (c) The least squares method is used to reduce the sum of errors in data points.
- (d) Less error given by least squares method shows that the model fits better.

3. Construct the simple linear regression equation of y on x that best fits the data (x_i, y_i) if

$$\sum_1^7 x_i = 113, \sum_1^7 x_i^2 = 1983, \sum_1^7 y_i = 182, \sum_1^7 x_i y_i = 3186.$$

4. Consider the set of points: $(1, 1), (-2, -1), (3, 2)$. Plot these points and the least-squares regression line in the same graph.
5. Consider the set of points:

x	0	1	2	3	4
y	2	3	5	4	6

Find the values of slope and y -intercept in the equation of least squares.

6. Write (i) $y = ax^b$ and (ii) $y = ae^{bx}$ (iii) $y = ab^x$ in a linear form $Y = A + BX$. What is $Y, X, A,$ and B .

- (a) Find the best values of a and b so that $y = ax^b$ fits the data given in the table:

x	80	40	20	10	5
y	333	375	422	475	533

(b) Find the best values of a and b so that $y = ae^{bx}$ fits the data given in the table:

x	2	4	6	8	10
y	4.077	11.084	30.128	81.897	222.62

(c) Find the best values of a and b so that $y = ab^x$ fits the data given in the table:

x	2	7	13	22	28
y	9	14	26	70	130

7. Consider the set of points:

$$(-1, 0), (0, 2), (1, 4), (k, 5).$$

The values of slope and y -intercept in the equation of least squares are 1.7 and 1.9 respectively. Determine the value of k .

8. Number of man-hours and the corresponding productivity (in units) are furnished below.

Man hours	3.6	4.8	7.2	6.9	10.7	6.1	7.9	9.5	5.4
Productivity	9.3	10.2	11.5	12	18.6	13.2	10.8	22.7	12.7

Fit a simple linear regression equation applying the method of least squares.

- Compute reasonably accurate values for the exponential function e^t for values of t lying in the interval $0 \leq t \leq 1$ by approximating it by a quadratic polynomial $p(t) = \alpha + \beta t + \gamma t^2$ using three and five equally spaced sample points.
- Determine the least-squares approximation of the type $ax^2 + bx + c$, to the function 2^x at the points $x_i = 0, 1, 2, 3, 4$.
- Obtain the least squares polynomial approximation of degree one and two for $f(x) = \sqrt{x}$ on $[0, 1]$.
- Obtain a linear polynomial approximation to the function $f(x) = \sin x$ on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ using the least squares approximation.
- Fit the following data with an unweighted least squares lines and weighted least squares lines with given weights w_i :

x	0	1	3	6
y	2	3	7	12
w_i	3	2	1/2	1/4