Subject : **Numerical Methods** (MA221)
Course Instructor(s): SK+GR+PD
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Problem Set 1 13.01.25 AY 2024-25 (Even)

Introduction and errors

1. What is the significant digits for the following numbers
   (a) 2.05×10^{-3}
   (b) 9.762×10^{4}
   (c) 1728.00
   (d) 0.0580730
   (e) $2.33 \times 6.085 \times 2.1$
   (f) $(4.52 \times 10^{-4}) \div (3.980 \times 10^{-6})$
2. Round off the following numbers to 4 significant digits:
   2.34567, 2.3455, 2.34449, 1.47383, 1473.27, 0.00276, 0.0027657
3. Calculate the value of $\sqrt{626} - \sqrt{625}$ correct to 4 significant digits.
4. Define Floating-Point Form, errors, truncation and rounding error.
5. Write the floating point representation of the following real numbers:
   (a) 6.238;
   (b) -0.0014.
6. Round off the numbers 865250 and 37.46235 to four significant digits and compute absolute, relative and percentage error in each case.
7. Find 0.348 + 0.1834 + 435.4 + 235.2 + 11.75 + 9.27 + 0.0849 + 0.0214 + 0.000354, assume that all their digits being correct. Find maximum, absolute and relative error in the sum also.
8. Suppose that you have the task of measuring the lengths of a bridge and a rivet and come up with 9999 and 9 cm, respectively. If the true values are 10,000 and 10 cm, respectively, compute (a) the true error and (b) the true percent relative error for each case.
9. If $x = 5.675, y = 4.737$ and $z = 4.373$, calculate $x(y-z)$ and $xy-yz$, to four significant digits, which one is more accurate.
10. Compute $y = x^3 \sin(x)$ for $x = \sqrt{2}(\approx 1.414)$. Determine absolute and relative error in y.
11. If $u = \frac{4xy^2}{z^3}$ and error in x, y, z be 0.001, compute the maximum relative error in u when $x = y = z = 1$.
12. Find $u = \log_e(x_1 + x_2^2)$, $x_1 = 0.97, x_2 = 1.132$. Obtain absolute and relative error in u.

13. Calculate the Round off error of $\frac{1}{3}$ and $\sqrt{2}$.

14. Use the decimal format with six significant digits (apply rounding at each step) for the following functions to calculate
    (a) $f(x) = \frac{1-\cos(x)}{\sin(x)}$ for $x = 0.007$,
    (b) $f(x) = \frac{\sqrt{(x+4)}-2}{x}$ for $x = 0.001$,
    (c) $f(x) = \frac{e^x-1}{x}$ for $x = 0.005$.

    the true relative error, due to rounding, in the value.

15. Evaluate the following polynomials at $x = 1.37$. Use 3-digit arithmetic with chopping. Evaluate the percent relative error.
    (a) $y = x^3 - 5x^2 + 6x + 0.55$.
    (b) $y = ((x - 5)x + 6)x + 0.55$.

    Compare the error for both the polynomials.

16. Use 5-digit arithmetic with chopping to determine the roots of the equation $x^2 - 5000.002x + 10$. Compute percent relative errors for your results.

17. Find the truncation error around $x = 0$ for the following functions
    (a) $f(x) = \sin x$,
    (b) $f(x) = \cos x$,
    (c) $f(x) = e^x$.

18. Using Taylor series expansion of the function $f(x) = e^x$, calculate the value of e^{-2} for the following cases.
    (a) Use the first four terms.
    (b) Use the first six terms.
    (c) Use the first eight terms.

    Use decimal numbers with six significant numbers (apply rounding at each step). In each case calculate also the true relative error.

19. In the Taylor's series expansion of e^x at $x = 0$. How many terms it would require to get an approximation of e^1 within a magnitude of true error of less than 10^{-6}.

20. Let $f(x) = \sqrt{1+x}$, $x \in [0, \frac{1}{2}]$. Find the second order Taylor polynomial (computed about 0) approximation to f and the corresponding error bound.

21. Let $p_n(x)$ be the Taylor polynomial of degree n to the exponential function e^x, $x \in [0, 1]$ such that the error for this approximation is less than 10^{-6}, that is,

    $$|e^x - p_n(x)| < 10^{-6} \quad \text{for all } x \in [0, 1].$$

    Then find n.

22. Given a value of $x_A = 2.5$ with an error of 0.01. Estimate the resulting error in the function $f(x) = x^3$.

23. Write the number 0.2 in binary form with sufficient number of digits so that the true relative error is less than 0.01.