23CS3068

Department of Mathematical Sciences

RAJIV GANDHI INSTITUTE OF PETROLEUM TECHNOLOGY, JAIS

Mid Semester Examination (AY 2024 - 25 / Even / Sem - IV)

NUMERICAL METHODS (MA 221)

Duration : 2 hours | 28 - Feb - 2025 | Total Marks : 30

■ **Instructions.**

i) Answer all the questions.

ii) Answer all parts of a question together.

iii) Use of **own** calculator is allowed.

iv) Symbols have their usual meanings.

1. Label the following statements (i)–(vi) as **TRUE/FALSE** only, no justification is needed. Each correct answer carries 1 mark and **(−1) marks for wrong answer.** *(1 × 6)*

   i) The relative error is closely related to the number of significant digits of an approximate number.

   ii) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ is a positive definite matrix.

   iii) If $\|H\| \nless 1$, then the iteration scheme $x^{(k+1)} = Hx^{(k)} + c$ is not convergent.

   iv) For solving a system of linear equations by Jacobi's method, as soon as a new value of a variable is found, it is used immediately in the iteration.

   v) The Regula Falsi iteration method is always convergent.

   vi) The curve obtained through the least square approximation always attains all the given data points.

2. (a) Construct a Taylor polynomial approximation (about 0) that is accurate to within 10^{-3} for the function $f(x) = \sin x$ in $[0, \pi]$. **(4)**

   (b) We can compute e^{-x} using Taylor's theorem, either using $e^{-x} \approx 1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \cdots$ or using $e^{-x} \approx \frac{1}{1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\cdots}$. Which approach is more susceptible to rounding error and why? **(2)**

3. (a) Show that the invertible matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$ is not LU decomposible. Do a suitable interchange of rows to this matrix, which has an LU factorization. **(5)**

   (b) Explain why the Gauss-Seidel iteration scheme is convergent for the following system of equations **(1+3)**

$$\begin{aligned} 4x + 2z &= 4 \\ 5y + 2z &= -3 \\ 5x + 4y - 10z &= 2. \end{aligned}$$

   Set up the Gauss-Seidel iteration scheme in matrix form.

   (c) Obtain an approximation in the sense of the principle of least squares in the form of a polynomial of the degree 2 to the function $\frac{1}{1+x^2}$ in the range $-1 \le x \le 1$. **(4)**

4. (a) Let a be a real number such that $0 < a \le 1$. Let $\{x_n\}_{n=1}^{\infty}$ be the iterative sequence of the Newton-Raphson method to solve the nonlinear equation $x - e^{-ax} = 0$. If x^* denotes the exact root of this equation and $x_0 > 0$, then show that **(4)**

$$(x^* - x_{n+1}) = \frac{1}{2}(x^* - x_n)^2.$$

   (b) Write down the orders of convergence in Newton-Raphson method for solving $f(x) = 0$ in the case of simple root and multiple root. **(1)**

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