Few examples of Pattern where We get something like 0 1 2 - - X $\boxed{\text{Non Zero}}$

P = A B A C A B A D A B A C A B A C
0 0 1 0 1 2 3 0 1 2 3 4 5 6 7 $\boxed{4}$ non-zero

P = A B A C A B A B A (by Anand Sagar)
0 0 1 0 1 2 3 $\boxed{2}$ 3

P = A B A C A B A B (by Uzair)
0 0 1 0 1 2 3 $\boxed{2}$

---

* Given a Pattern $P[1, 2, \_\_\_ m]$ of length m, obtain $\pi[1, 2, \_\_ m]$

$\pi[i]$ $1 \le i \le m$ = Length of the longest proper prefix which is also a proper suffix for pattern $P[1, 2, -- i]$

Naive Approach

For Pattern $P[1, 2, -- i]$
* Find all proper prefix
* Find all proper suffix
* Take one which is common and
* have max. length
* This max length is $\pi[i]$

Example

$p = $ | a | b | a | b | a | c | a |

$\pi[1]$ $P[1] = a$

Proper prefix: ϕ Proper suffix: ϕ

Hence $\pi[1] = 0$

$\pi[2]$ $P[1,2] = ab$

Proper prefix: a Proper suffix: b

No common so $\pi[2] = 0$

$\pi[3]$ $P[1,2,3] = aba$

Proper prefix	Proper suffix
a	a
ab	ba

Common is $\boxed{a}$ so $\pi[3] = 1$

$\pi[4]$ $P[1,2,3,4] = abab$

proper prefix	Proper suffix
a	b
ab	ab
aba	bab

Common is $\boxed{a}$ & $\boxed{ab}$ so $\pi[4] = 2$

$\underline{\pi[5]}$ $P[1,2,3,4,5]$ = ababa

Proper prefix	Proper suffix
a	a
ab	ba
aba	aba
abab	baba

Common is $\boxed{a}$ and $\boxed{aba}$ so $\pi[5] = 3$

$\underline{\pi[6]}$ $P[1,2,3,4,5,6]$ = ababac

Proper prefix	Proper suffix
a	c
ab	ac
aba	bac
abab	abac
ababa	babac

No common so $\pi[6] = 0$

$\underline{\pi[7]}$ $P[1,2,3,4,5,6,7]$ = ababaca

Proper prefix	Proper suffix
a	a
ab	ca
aba	aca
abab	baca
ababa	abaca
ababac	babaca

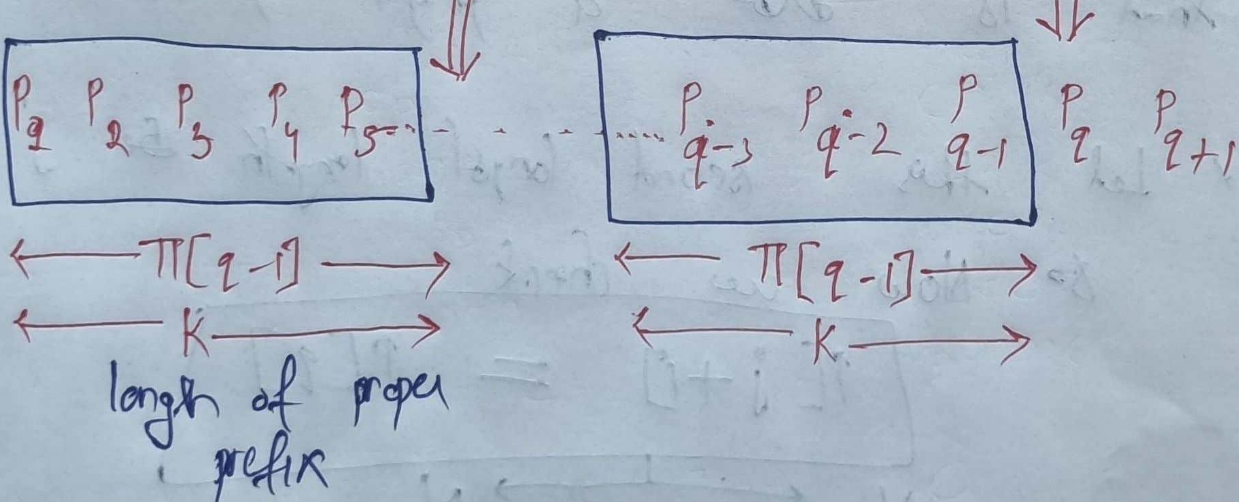
Common is $\boxed{a}$ so $\pi[7] = 0$

$$\pi[i+1] \leq \pi[i]+1$$

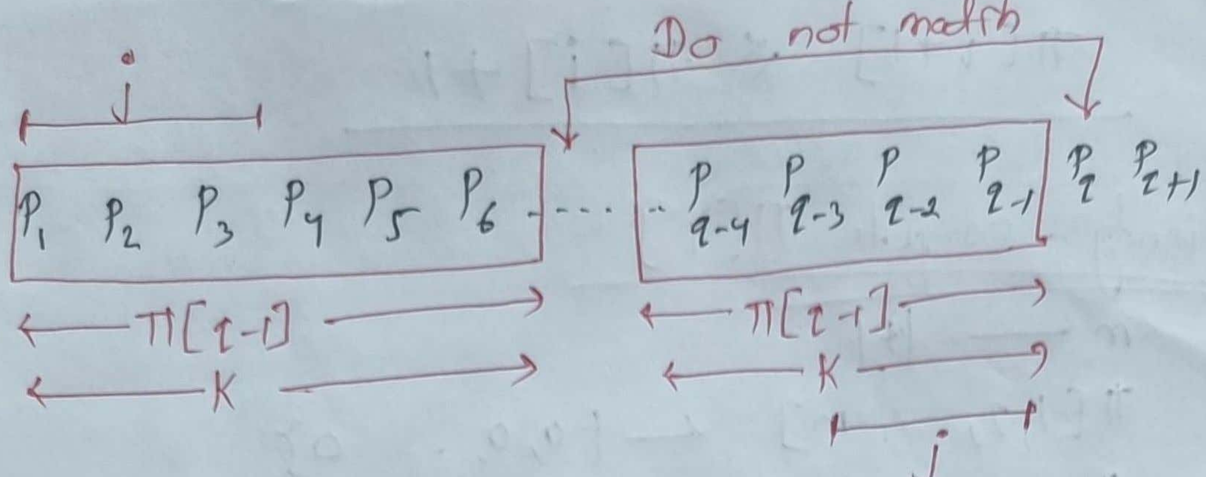
Efficiently compute $\pi[i]$

```
m ← |P|
π[1, 2, ... m] ← {0, 0, ... 0}
k ← 0            ▷ Denotes the length of proper prefix

FOR q ← 2 to m   ▷ Find π[q]
    while k > 0 and P[k+1] ≠ P[q]
        k ← π[k]
    if P[k+1] = P[q]
        k ← k+1
    π[q] ← k
```



$$\boxed{\begin{array}{l} \text{If } P[k+1] = P[q] \\ \qquad \pi[q] \leftarrow k+1 \end{array}}$$



$P[K+1] \neq P_q$ here

* K is the length of the longest proper suffix which is ending at P_{q-1} and this a also a proper prefix.

* here $P[K+1] \neq P_q$ so we need to find the second longest proper suffix which is ending at P_{q-1}. and that is also a proper prefix.

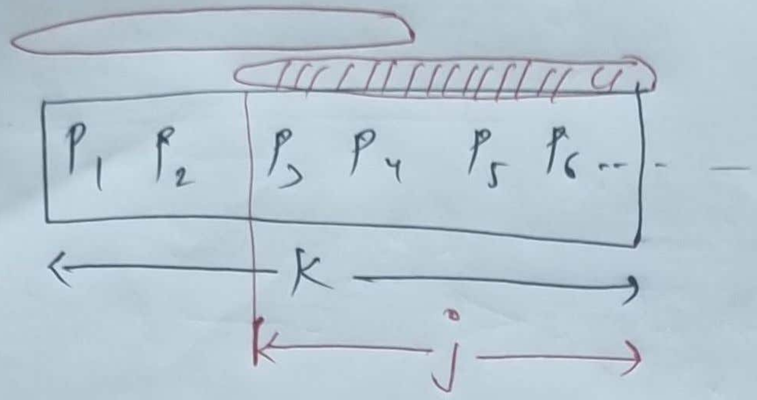
* Let this second longest length be j

so Now we check

$$\boxed{P[j+1] = P[q]}$$

Yes ⟵ ⟶ No

How to find j



So $j = \pi[k]$