0-1 Knapsack Problem

* N items (can be same or different)
* i^{th} item worth V_i and weighs w_i
* Must leave or take each item

Fractional Knapsack Problem

* N items (can be same or different)
* i^{th} item worth V_i and weighs w_i
* Can take fractional part of each item

Greedy do not work for 0-1

Item	A	B	C	D
Price	12	9	9	5
Weight (Size)	24	10	10	7

$W = 25$

Optimal: B+C Weight = 20, Price = 18

(i) Greedy: Largest price first A Weight = 24, Price = 12

(ii) Greedy: Smallest size first (Weight) D+B Weight = 17, Price = 14

2

Item	A	B	C
Price	50	140	60
Size/weight	5	20	10
Ratio	10	7	6

$W = 30$

Greedy: Take largest Ratio

A + B

Weight = 5 + 20 = 25

Price = 50 + 140

= 190

optimal B + C

Weight = 20 + 10 = 30

Price = 140 + 60 = 200

Greedy for Fractional

$A + B + \frac{1}{2}C$

Weight = 5 + 20 + 5 = 30

Price = 50 + 140 + 30 = 220

Fractional knapsack

* For each object i, suppose a fraction $0 \leq x_i \leq 1$ can be placed in knapsack, then profit earned is $v_i x_i$. Objective is to maximize profit subject to capacity constraint

$$\text{Maximize} \sum_{i=1}^{n} v_i x_i \quad \text{——} \ (1)$$

$$\text{Subject to} \sum_{i=1}^{n} w_i x_i \leq W \quad \text{——} \ (2)$$

$$\text{where } 0 \leq x_i \leq 1, \quad v_i > 0 \text{ and } w_i > 0 \quad \text{——} \ (3)$$

* Feasible solution is any subset $\{x_1, x_2, ---, x_n\}$ subject to (2) and (3)

* Optimal solution is feasible solution that maximize (1)

1) ~~Greedy : Sort in non-increasing order of Values/Price~~ (1)

~~Decreasing~~

Item	A	B	C
Value	20	16	10
Wt.	14	6	8

$W = 19$

~~Order → A, B, C~~

~~Solution~~ $A + \frac{5}{6} B$ ⇒ $20 + \frac{5}{6} \cdot 16$

14

$20 + 13.33 = 33.33$

1) Greedy Largest Price First

Item	A	B	C
Value / Price	25	24	15
Weight	18	15	10

$W = 20$

Order $A \to B \to C$

Solution $A + \frac{2}{15} \cdot B$

Value $= 25 + \frac{2}{15} 24 = 28.2$

2) Greedy : Smallest Weight First

Order : $C \to B \to A$

Solution $C + \frac{2}{3} \cdot B$

Value = $15 + \frac{2}{3} \cdot 24 = 31$

3 Greedy : Fractional (Largest Ratio of Value/Weight)

$A = \frac{25}{18}$ $B = \frac{24}{15}$ $C = \frac{15}{10}$

Order = $B \to C \to A$

Solution $B + \frac{1}{2} C$

Value = $24 + \frac{1}{2} \cdot 15 = 31.5$

Greedy-Fractional-knapsack(w, v, W)

1) For $i \leftarrow 1$ to n

$x[i] \leftarrow 0$

2) $Weight \leftarrow 0$

3) For $i \leftarrow 1$ to n

if $Weight + w[i] \leq W$

$x[i] \leftarrow 1$

$Weight \leftarrow weight + w[i]$

else

$$x[i] = \frac{(W - weight)}{w[i]}$$

$weight \leftarrow W$

BREAK

4) Return x

Can Calculate $\underline{\underline{\sum_{i=1}^{n} x[i] \cdot v[i]}}$ <u>Now</u>

<u>In this case</u>

(i) largest price first
(ii) Smallest weight first
(iii) largest $\frac{Value}{Weight}$ First

} Can be used

Complexity Analysis

(i) Sorting $O(n\log n)$
(ii) Remaining $O(n)$

$\Big\rangle$ $O(n)$

If create Heap $\frac{v_i}{w_i}$
then $O(n)$ to create Heap
And Max n time Heapify

Total $O(n) + O(n\log n) = O(n\log n)$

But if knapsack is filled in less item
then $O(n) + O(c \cdot \log n)$
$= O(n)$ ⟶ Constant