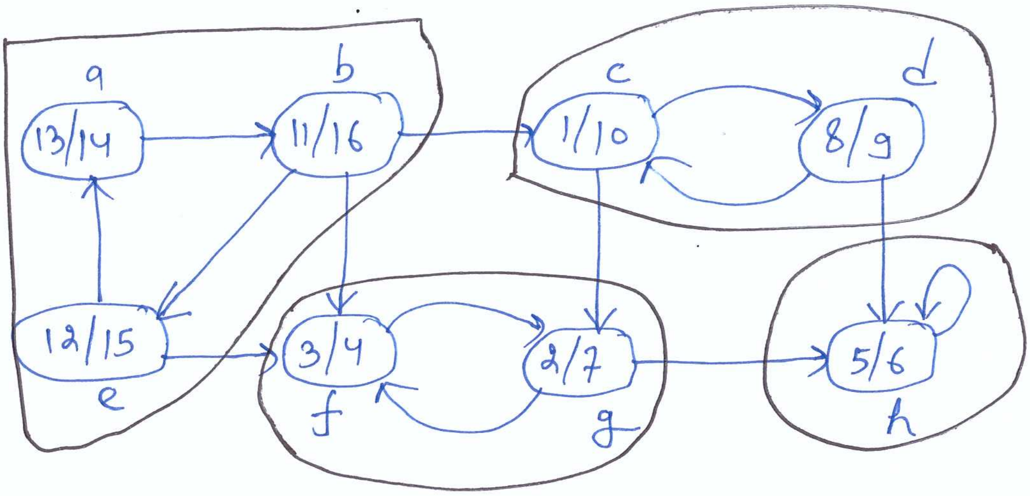
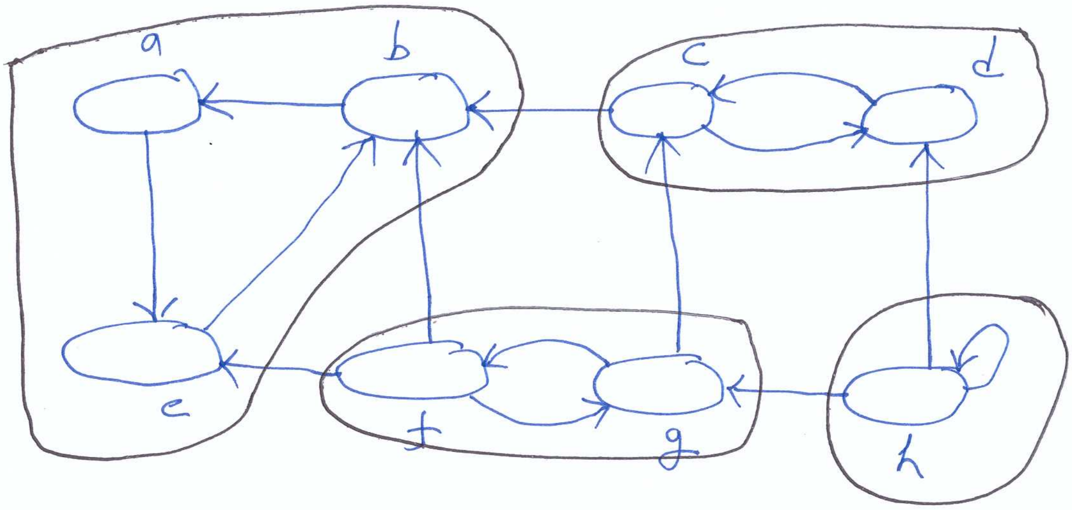
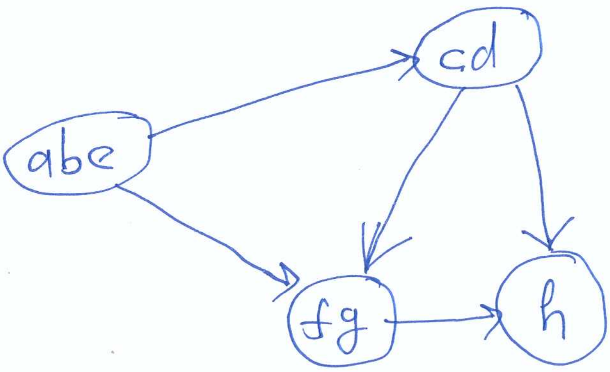
* A strongly connected component of a directed graph G = (V, E) is a MAXIMAL set of vertices such that every pair of vertex in this set is Reachable from each other





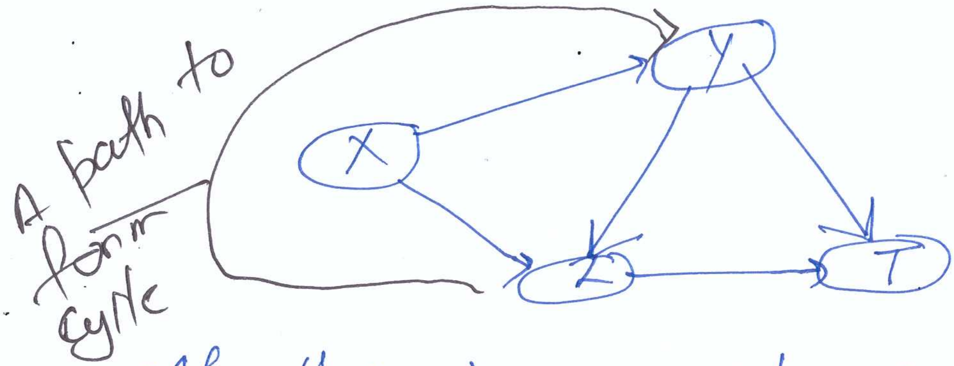


Component Graph

Observation

* Graph G and its transpose G^T have exactly same strongly connected component.

* COMPONENT Graph is a DAG



Component Graph

* If there is a cycle in <u>Component Graph</u>

$Y \longrightarrow Z$ and $Z \rightsquigarrow Y$

means all nodes of Y and Z are Reachable from each other.

Thus $Z \cup Y$ is SCC But this is not TRUE. Hence <u>DAG</u>

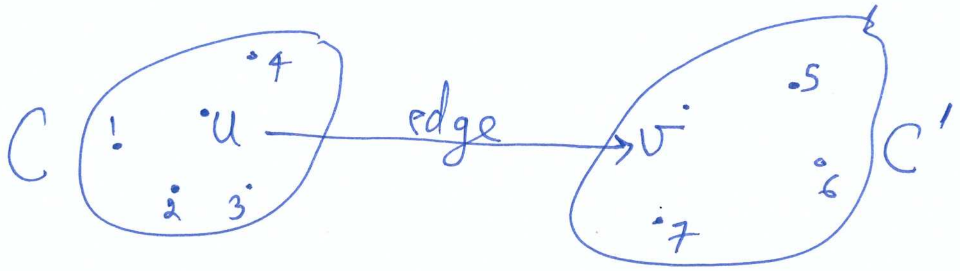
Let $U \subseteq V$

$d(U) = \text{Min}_{u \in U}\{u.d\} \rightarrow$ Earliest discovery time

$f(U) = \text{Max}_{u \in U}\{u.f\} \rightarrow$ Latest finishing time

⇒ Let C and C' be distinct SCC in $G=(V,E)$
Suppose there is an edge $(u,v) \in E$ such that $u \in C$ and $v \in C'$. then

$$f(C) > f(C')$$



<u>Case 1</u> $d(C) < d(C')$ <u>Case 2</u> $d(C) > d(C')$

<u>Case I</u> $d(C) < d(C')$

Let x be the first vertex discovered in C, i.e, $x.d = d(C)$

$x \rightsquigarrow u \rightarrow v \rightsquigarrow w$ — $\boxed{w \in C'}$

There exist a path of WHITE VERTICES from x to all vertices in C & C'

So x has the latest finishing time of any of its descendants so $\boxed{x.f = f(C) > f(C')}$

Case II $d(C) > d(C')$

$\boxed{\text{Will use DAG property of Component Graph}}$

Let y be the first discovered in C', i.e., $y.d = d(C')$.

* At time $y.d$, all vertices in C' are WHITE and G contains a path from y to each vertex in C' consisting of white vertices.
* All vertex in C' are descendant of y

$y.f = f(C')$

* No vertex in C is Reachable from y or any node in C' [Else component graph will not be DAG]
* At time $y.f$ all vertices in C are white

Thus for any $w \in C$ we have $w.f > y.f$

i.e, $f(C) > f(C')$

⑤

⇒ Let C and C' be distinct SCC in $G=(V,E)$

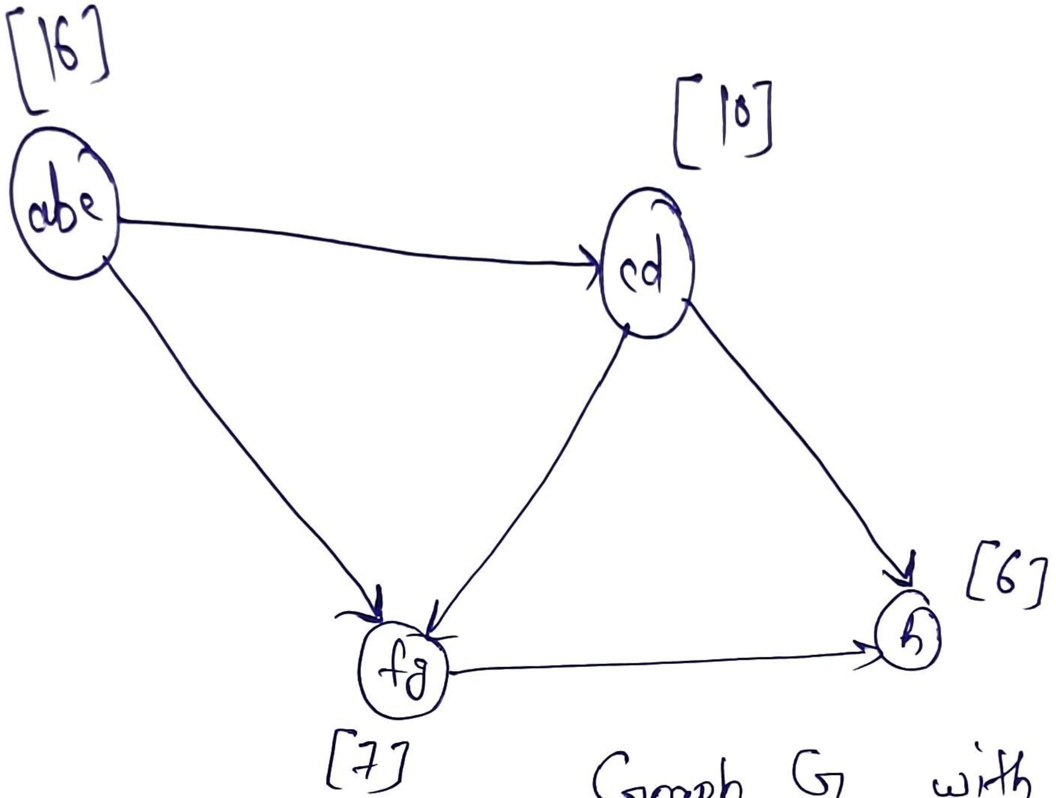
Suppose there is an edge $(u,v) \in E^T$ where $u \in C$ and $v \in C'$ then

$f(C) < f(C')$

Since $(u,v) \in E^T$ means $(v,u) \in E$

SCC of G and G^T are same

$f(C) < f(C')$

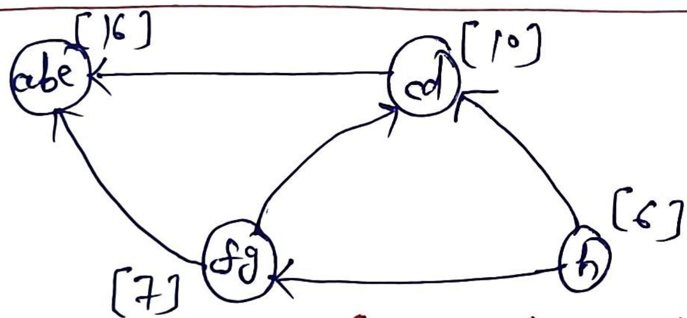


Graph G with E

$f(C) > f(C')$

if $u \longrightarrow v$

$u \in C$ $\quad v \in C'$



Graph G with E^T

$f(C) < f(C')$

if $u \longrightarrow v$

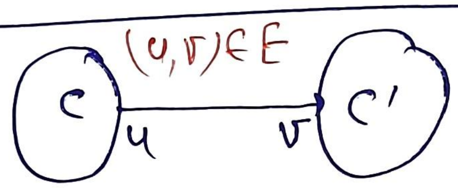
$u \in C$ $\quad v \in C'$

DFS on G^T

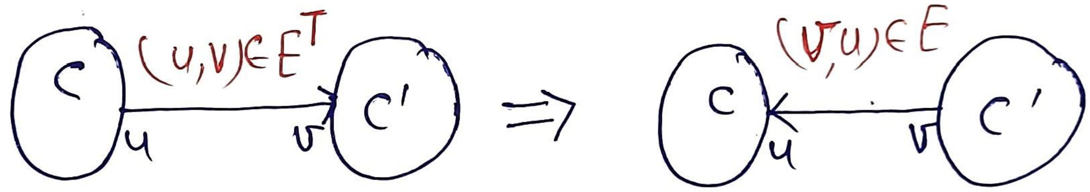
* Start this DFS with the component C (basically a vertex with maximum finish time based on DFS(G) in that component) whose finish time is maximum. $\underline{\text{Max}}$ $f(C)$

$\Downarrow$

* This will starts DFS from x and visit all vertices in G

I) 

$f(C) > f(C')$

II) 

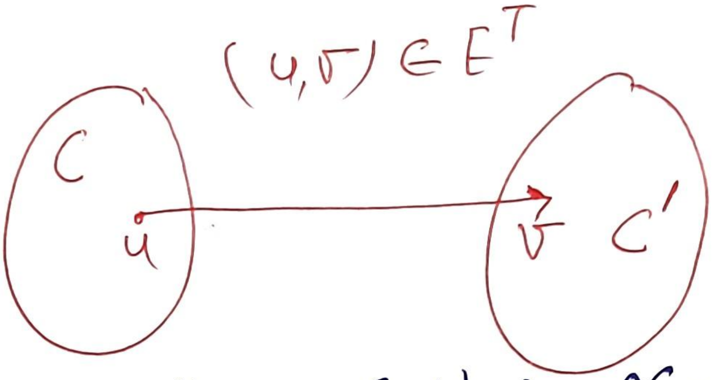
$f(C') > f(C)$

WHY

By (II), in G^T there cannot be any edge from C to any other strongly connected component. Hence the search from $\boxed{x}$ will not visit vertices in any other component.

Let us See WHY (II) is saying this

in G^T



then $f(C') > f(C)$

However in our case $f(C)$ is maximum in the graph G.

~~Hence no~~

$f(C) < f(C')$

the Component whose finish time is greater [(C') here] is receiving the edge. edge is not going out.

Only for WHY

Now continue rest of the discussion

* Tree rooted at x contains all vertices in C.

Once all vertices in C are visited, We select a vertex(y) from another component C'
[This y has max. finish time among all vertices except vertices in C]

* Now again y will discover all nodes in C' but because of (II) ~~only~~ from y only the nodes of C' are traversed. Now edge can be there from C' to C but as C has already been visited so no use.

In general, edge can be present from the current component to already obtained component in G^T.