Application of DFS on Directed Graph

1

I/P → Given a directed graph G

O/P: G is Strongly connected or not ??

Simple Approach

1. For each $u \in V$

   DFS starting from u

2. If all DFS traversal gives all vertices means there is a path from all vertices to all other vertices and Hence STRONGLY CONNECTED

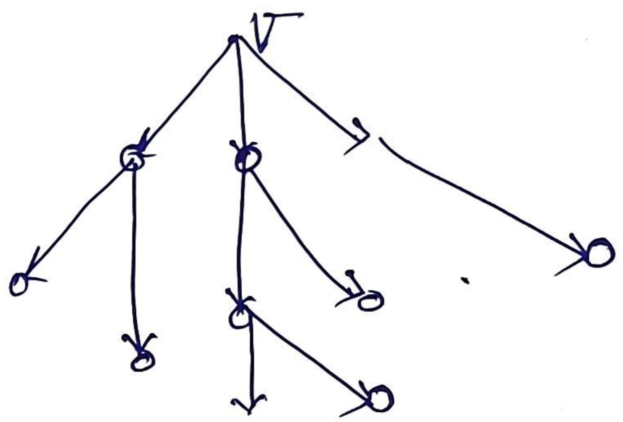
$$\text{Time} = |V|\,(|V| + |E|)$$

$$= n(m+n)$$

$$= O(mn)$$

Better Approach

i) If DFS(v) visits all vertices in G then $\exists$ a path from v to all other vertices in G.



ii) If we know $\exists$ a path from every vertex in G to v

This is TRUE (Assume)

(i) $\wedge$ (ii) $\Longrightarrow$ Strongly connected G

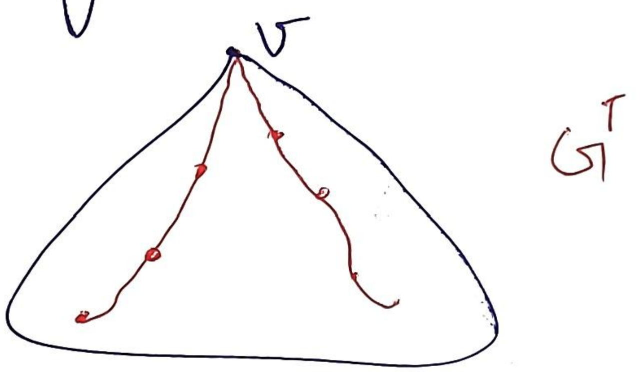
(x, y) is pair of vertex — then $x \rightsquigarrow v$ (ii) then $v \rightsquigarrow y$ (i)

How do we check that if there is a path from every vertex to v ??

$$G \xrightarrow[\text{edges}]{\text{Reverse}} G^T \text{ (Transpose of } G\text{)}$$

(i) Do DFS on a vertex v in G^T, i.e., DFS(v) in G^T

→ If all vertices are visited in G^T by v then



Here in G^T, we are reaching all vertices from v. It means in Original graph G, there is a path from every vertex to v.

If any vertex is not visited in DFS(v) in G^T then can we say that graph is not strongly connected !!

Yes

Time Complexity

$2(|V|+|E|)$

1) 2 DFS
2) Reverse edge $|E|$

$O(|V|+|E|) = O(m+n)$

DFS in G^T $\Rightarrow$ Consider incoming edges

DFS in G $\Rightarrow$ Consider outgoing edges.

Algorithm

1) Pick an arbitrary vertex $v \in V$
2) Do DFS(v) in G
3) Reverse edges in G and get G^T
4) Do DFS(v) in G^T
5) If all vertices are visited in both DFS
   G is strongly connected

   Else
   Not strongly connected