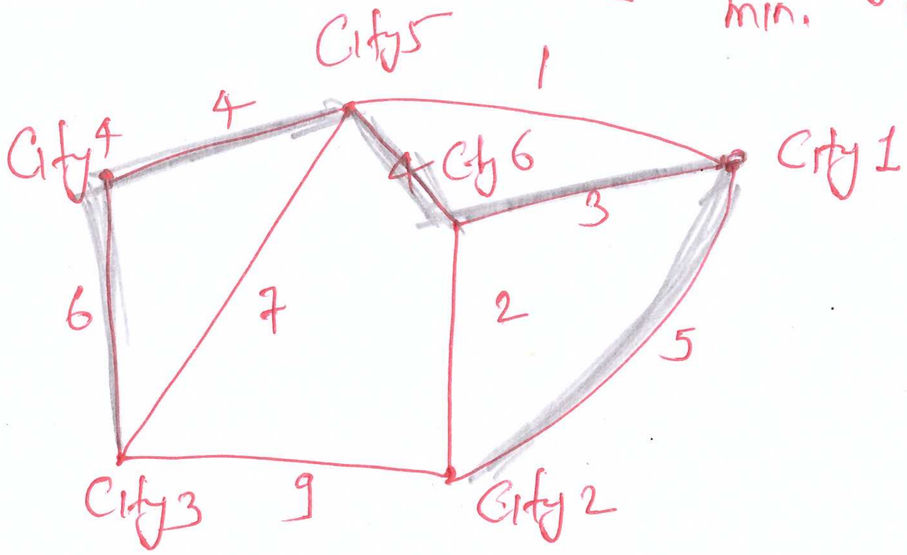
Spanning Tree

- Connected Subgraph w/o Cycle
- Should include all vertices

Spanning Tree has $|V|-1$ edges

Minimum Spanning Tree : Spanning term of min. weight



length of Spanny Tree (ST) = Sum of the length of the edgs in th Tree

length of Tree = 6+4+4+3+5 = 22

weight = ω

$$\omega(T) = \underbrace{\sum_{(u,v)\in T} \omega(u,v)}_{\text{Should be minimum}}$$

KRUSKAL Algo

(1) Sort edges in increasing order of length
$e_1 \quad e_2 \text{ ------- } e_m$

(2) $T \leftarrow \phi$

(3) for each edge $(u,v) \in E$

if $(u,v) \cup T$ is a Tree

$T \leftarrow T \cup \{(u,v)\}$

(4) Return T

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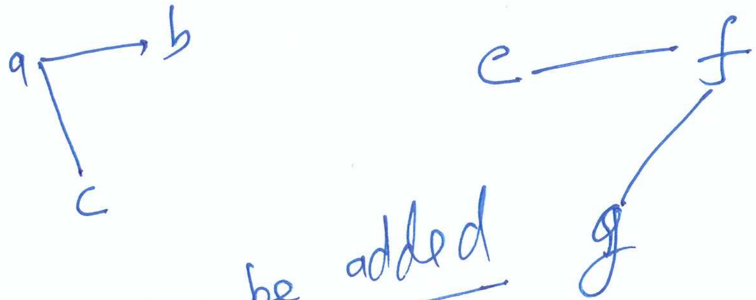
* Initially each vertex is COMPONENT
* For each edge (u,v); check whether u and v are in same/ diff. COMPONENT
* If Same COMPONENT $\longrightarrow$ CYCLE
  Else No cycle and add edge in MST



b,c cannot be added

b,e can be added

KRUSKAL

Kroskal $g_1 < g_2 \cdots < g_{i-1} < g_i < g_{i+1} \cdots < g_{n-1}$

Optimal $f_1 < f_2 \cdots < f_{i-1} < f_i < f_{i+1} \cdots < f_{n-1}$

Case I

$$\begin{array}{ccccccccc}
g_1 < & g_2 & \cdots < & g_{i-1} & \boxed{< g_i} & \cdots & < g_{n-1} \\
\| & \| & & \| & \vee & & \\
f_1 < & f_2 & \cdots < & f_{i-1} & \boxed{< f_i} & \cdots & < f_{n-1}
\end{array}$$

length $(f_i) <$ length (g_i)

* Edge f_i is distinct from $g_1, g_2, \ldots g_{i-1}$ and $f_i < g_i$ then WHY KROSKAL HAD NOT ADDED f_i to MST?

Becoz $f_i \cup \{g_1, g_2, \ldots g_{i-1}\}$ contains CYCLE

$= f_i \cup \{f_1, f_2, \ldots \cancel{g} f_{i-1}\}$

Thus OPTIMAL ST HAS CYCLE which is NOT POSSIBLE

Case II

$$\begin{array}{ccccccccccc} g_1 & < & g_2 & \cdots & < g_{i-1} & < & \boxed{g_i} & < & g_{i+1} & \cdots & < g_{n-1} \\ \| & & \| & & \| & & \wedge & & & & \\ f_1 & < & f_2 & \cdots & < f_{i-1} & < & \boxed{f_i} & < & f_{i+1} & \cdots & < f_{n-1} \end{array}$$

$\text{length}(g_i) < \text{length}(f_i)$

* As $g_i < f_i$ and $f_i < f_{i+1} \cdots < f_{n-1}$

so g_i cannot be from $f_i, f_{i+1}, \ldots, f_{n-1}$

* As $g_i > g_{i-1}$ so g_i cannot be from

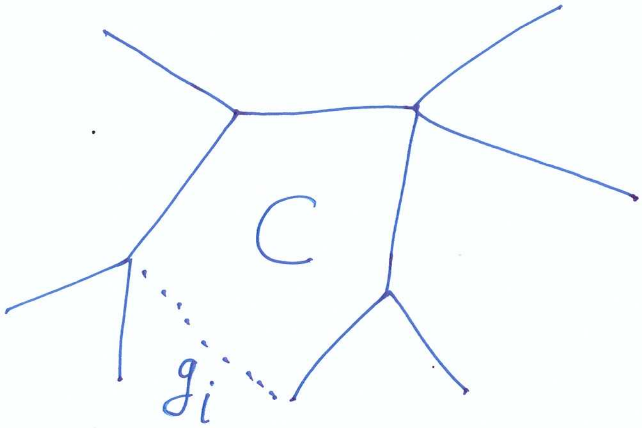
$$\underbrace{\underline{g_1, g_2, \ldots g_{i-1}} \text{ OR } \underline{f_1, f_2, \ldots f_{i-1}}}_{\text{ARE SAME}}$$

Thus, g_i cannot be from $\underline{f_1, f_2, \ldots, f_{n-1}}$

(i) Add g_i to optimal Tree

(ii) After adding g_i in optimal tree, we will get a Cycle. Let $\boxed{C}$ be CYCLE

Case II.A



→ If any edge in this cycle is larger then g_i THEN

→ Add g_i
→ Remove the larger edge

→ NOW the whole cost of TREE will Reduce. BUT This is NOT POSSIBLE AS TREE is OPTIMAL

⇓

CONTRADICTION

Case II-B

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* All edge in cycle $\boxed{C}$ have length less than g_i.

* But if this is the case then all edge of cycle should be from

$\boxed{f_1, f_2, \text{——} f_{i-1}}$

becoz $f_i, f_{i+1}, f_{i+2}, \text{——} f_{n-1}$ are longer than g_i

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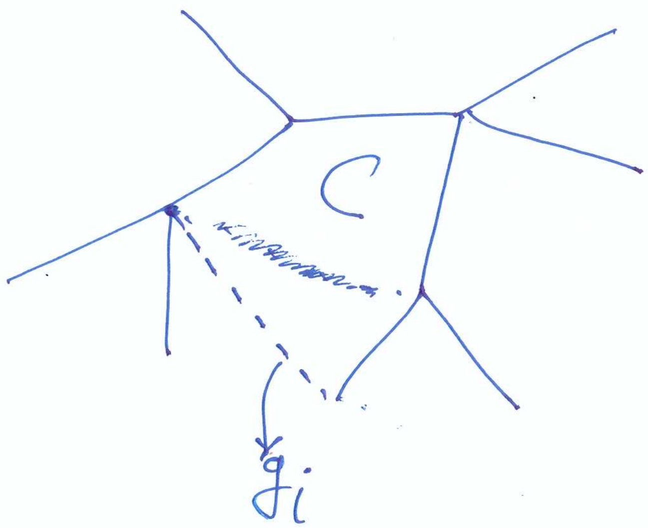

$f_1 \text{——} f_{i-1}$ means $g_1 \text{——} g_{i-1}$

· Means KRUSKAL ALSO had cycle

· BUT THIS CANNOT HAPPEN