Let $s, v_2, v_3, \ldots v_n$ be then vertices in ORDER taken from Heap by Algo.

When v_i is Removed show that $d[v_i] = \delta[v_i]$

Proof

Claim is TRUE for source node s.
When s is Removed $d[s] = \delta(s) = 0$
because All Edge weights are POSITIVE.

Assume that the claim is TRUE for first $k-1$ Nodes when they are deleted.

i.e,

$$d[v_i] = \delta[v_i]$$

for $i = 1, 2, \ldots, k-1$

* Now focus on the situation when v_k is Removed.

* As per Algo

$$d[v_k] \leq d[v_{k+1}]$$
$$\leq d[v_{k+2}]$$
$$\vdots$$
$$\leq d[v_n]$$

#1

If the shortest path from s to v_k has only vertices from $R = \{s, v_2, v_3, \ldots v_{k-1}\}$ then

$$d[v_k] = \delta(v_k)$$

#2

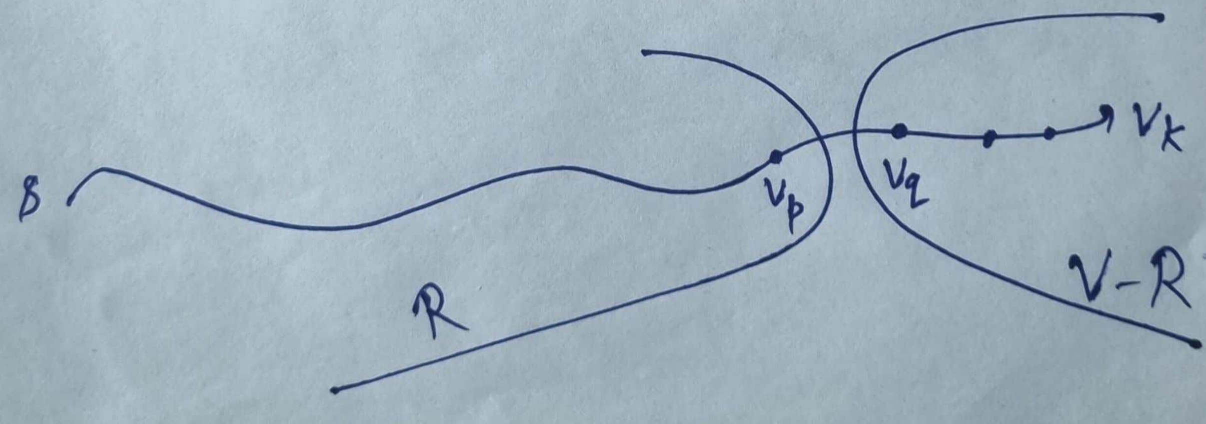
Assume that $d[v_k] > \delta(v_k)$

It follows that the shortest path from s to v_k involves the vertices in the set $V-R$.

* Let the first vertex $v_q \in V-R$ on the shortest path from s to v_k.

* Let v_p denote the vertex before v_q ~~to~~ on the shortest path from s to v_k.

$v_p \in R$ and $v_q \in V-R$



When v_p is deleted, all its edges are RELAXED including the edge (v_p, v_q) and thus

$$d[v_q] = \delta(v_q)$$

* Since all edges are +ve edges, $\delta(v_q) < \delta(v_k)$ and hence $d[v_q] < d(v_k)$

But it means that v_k could not have been chosen before v_q by Algo, contradicting the choice of v_k as a vertex for which $d[v_k] < \delta(v_k)$ when v_k is deleted from Heap.