$$\left[\begin{array}{cc|cc} 2 & 1 & 3 & 2 \\ 2 & 1 & 3 & 1 \\ \hline 1 & 3 & 2 & 4 \\ 4 & 1 & 2 & 3 \end{array}\right] \left[\begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \\ \hline 3 & 4 & 2 & 1 \\ 1 & 2 & 4 & 3 \end{array}\right] = \left[\begin{array}{cc|cc} 17 & 23 & 22 & 18 \\ 16 & 21 & 18 & 15 \\ \hline 23 & 27 & 29 & \cancel{28}\,21 \\ 17 & 25 & 30 & 28 \end{array}\right]$$

$$C_{11} = A_{11}B_{11} + A_{12}B_{21}$$

$$= \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 7 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 11 & 16 \\ 10 & 14 \end{bmatrix} = \begin{bmatrix} 17 & 23 \\ 16 & 21 \end{bmatrix}$$

$$C_{12} = A_{11} \cdot B_{12} + A_{12}B_{22}$$

$$= \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 9 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} 14 & 9 \\ 10 & 6 \end{bmatrix} = \begin{bmatrix} 22 & 18 \\ 18 & 15 \end{bmatrix}$$

$$C_{21} = A_{21}B_{11} + A_{22}B_{21}$$

$$= \begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} 8 & 13 \\ & \end{bmatrix}} \begin{bmatrix} 13 & 11 \\ 8 & 11 \end{bmatrix} + \begin{bmatrix} 10 & 16 \\ 9 & 14 \end{bmatrix} = \begin{bmatrix} 23 & 27 \\ 17 & 25 \end{bmatrix}$$

$$C_{22} = A_{21}B_{12} + A_{22}B_{22}$$

$$= \begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 7 \\ 14 & 17 \end{bmatrix} + \begin{bmatrix} 20 & 14 \\ 16 & 11 \end{bmatrix} = \begin{bmatrix} 29 & 21 \\ 30 & 28 \end{bmatrix}$$

$$P1 = A_{11}(B_{12} - B_{22})$$

$$= \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}\left(\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\right) = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 0 & 4 \end{bmatrix}$$

$$P_2 = (A_{11} + A_{12})B_{22}$$

$$= \left(\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 3 & 1 \end{bmatrix}\right)\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 22 & 14 \\ 18 & 11 \end{bmatrix}$$

$$P_3 = (A_{21} + A_{22})B_{11}$$

$$= \left(\begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix}\right)\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 31 & 27 \\ 22 & 24 \end{bmatrix}$$

$$P_4 = A_{22}(B_{21} - B_{11})$$

$$= \begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix}\left(\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}\right) = \begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} -8 & 0 \\ -5 & 1 \end{bmatrix}$$

$$P_5 = (A_{11} + A_{22})(B_{11} + B_{22})$$

$$= \left(\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix}\right)\left(\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\right) = \begin{bmatrix} 4 & 5 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ 8 & 6 \end{bmatrix} = \begin{bmatrix} 52 & 42 \\ 44 & 36 \end{bmatrix}$$

$$P6 = (A_{12} - A_{22})(B_{21} + B_{22})$$

$$= \left(\begin{bmatrix} 3 & 2 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix}\right)\left(\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\right) = \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} -5 & -5 \\ -5 & -5 \end{bmatrix}$$

$$P7 = (A_{11} - A_{21})(B_{11} + B_{12})$$

$$= \left(\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix}\right)\left(\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & -2 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} -8 & -2 \\ -8 & -12 \end{bmatrix}$$

$C_{11} = P5 + P4 - P2 + P6$

$$= \begin{bmatrix} 52 & 42 \\ 44 & 36 \end{bmatrix} + \begin{bmatrix} -8 & 0 \\ -5 & 1 \end{bmatrix} - \begin{bmatrix} 22 & 14 \\ 18 & 11 \end{bmatrix} + \begin{bmatrix} -5 & -5 \\ -5 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 23 \\ 16 & 21 \end{bmatrix}$$

$C_{12} = P1 + P2$

$$= \begin{bmatrix} 0 & 4 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 22 & 14 \\ 18 & 11 \end{bmatrix} = \begin{bmatrix} 22 & 18 \\ 18 & 15 \end{bmatrix}$$

$C_{21} = P3 + P4$

$$= \begin{bmatrix} 31 & 27 \\ 22 & 24 \end{bmatrix} + \begin{bmatrix} -8 & 0 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 23 & 27 \\ 17 & 25 \end{bmatrix}$$

$C_{22} = P5 + P1 - P3 - P7$

$$= \begin{bmatrix} 52 & 42 \\ 44 & 36 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 31 & 27 \\ 22 & 24 \end{bmatrix} - \begin{bmatrix} -8 & -2 \\ -8 & -12 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & 21 \\ 30 & 28 \end{bmatrix}$$

Stressen

$$P1 = A_{11}(B_{12} - B_{22})$$

$$P2 = (A_{11} + A_{12})B_{22}$$

$$P3 = (A_{21} + A_{22})B_{11}$$

$$P4 = A_{22}(B_{21} - B_{11})$$

$$P5 = (A_{11} + A_{22})(B_{11} + B_{22})$$

$$P6 = (A_{12} - A_{22})(B_{21} + B_{22})$$

$$P7 = (A_{11} - A_{21})(B_{11} + B_{12})$$

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} P5 + P4 - P2 + P6 & P1 + P2 \\ P3 + P4 & P5 + P1 - P3 - P7 \end{bmatrix}$$

7 Multiplication

18 Add ~~Supp~~

Karatsuba Algorithm

$x = x1 * B^m + x2$

$y = y1 * B^m + y2$

B is the base where $m < n$

Multiply two n-digit no.

$xy = (x1 \times B^m + x_2)(y1 * B^m + y2)$

$= \underbrace{x1 * y1 * B^{2m}}_{a} + \underbrace{x1y2\, B^m + x2y1\, B^m}_{b} + \underbrace{x2y2}_{c}$

$= x1 \cdot y1 \cdot B^{2m} + (x1y2 + x2y1)\, B^m + x2y2$

$a = x1 \cdot y1$

$b = x1y2 + x2y1$

$c = x2y2$

$$\boxed{\Rightarrow xy = a \cdot B^{2m} + b \cdot B^m + c}$$

$X = 61\,438\,521$

$Y = 9473\,6407$

$XY = 5820\ 464\ 730\ 934\ 047$

$X = \{\ X_L = 6143 \quad X_R = 8521$

$X = X_L \cdot 10^4 + X_R$

$B = 10$

$m = 4$

$n = 8$

$Y = \{\ Y_L = 9473 \quad Y_R = 6407$

$Y = Y_L \cdot 10^4 + Y_R$

$$XY = X_L Y_L 10^8 + (X_L Y_R + X_R Y_L) 10^4 + X_R Y_R$$

placing 8 zero will take n time
Als addition will rqure n time
Both will componsate

$T(n) = 4T(n/2) + O(n) \Rightarrow T(n) = O(n^2)$

$$X_L Y_R + X_R Y_L = \underbrace{(X_L - X_R)(Y_R - Y_L) + X_L Y_L + X_R Y_R}_{\text{One multiplication}}$$

$T(n) = 3T(n/2) + O(n)$

$T(n) = O(n^{\log_2 3}) = O(n^{1.59})$

$X = 5678$ length = n

$Y = 1234$

$a = 56 \quad b = 78$

$c = 12 \quad d = 34$

$x = a \cdot B^{n/2} + b$

B = Base

$ac = 672$ — (1)

$bd = 2652$ — (2)

$(a+b)(c+d) = 134 \cdot 46 = 6164$ — (3)

$3 - 1 - 2 = 2840$

Addition of 3 n digit nums

max 2n-1

6720000

2652

Add 284000

7006652 = 1234 · 5678

$0 \leq x, y \leq \omega^n$

$x = x_1 \cdot \omega^{n/2} + x_0$

$y = y_1 \cdot \omega^{n/2} + y_0$

$(\omega = 2, 10)$ radix

x_1 = High Signif x_0 = Low

y_1 = High y_0 = Low

$0 \leq x_0, x_1 \leq \omega^{n/2}$

$0 \leq y_0, y_1 \leq \omega^{n/2}$