

# 1 Merge Sort with Four Sub-problems

Merge sort which divides the array of size  $n$  recursively into four sub-arrays of size  $n/4$  to sort it with a separate `Merge()` procedure which merges four sorted arrays.

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**Algorithm 1** MERGE-SORT( $A, \text{low}, \text{high}$ )

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1: if low < high then
2:   mid2 = low + (high-low)/2                                ▷ Middle point of the entire array
3:   mid1 = low + (mid2-low)/2                                ▷ Middle point of the array A[low...mid2]
4:   mid3 = (mid2 + 1) + (high-(mid2+1))/2                 ▷ Middle point of the array A[mid2 + 1...high]
5:   MERGE-SORT(A, low, mid1)
6:   MERGE-SORT(A, mid1 + 1, mid2)
7:   MERGE-SORT(A, mid2 + 1, mid3)
8:   MERGE-SORT(A, mid3 + 1, high)
9:   MERGE(A, low, high, mid1, mid2, mid3)                ▷ Merge four sorted sub-arrays
10: end if
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**Algorithm 2** MERGE( $A, \text{low}, \text{high}, \text{mid}_1, \text{mid}_2, \text{mid}_3$ )

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1:  $n_1 \leftarrow \text{mid}_1 - \text{low} + 1$ 
2:  $n_2 \leftarrow \text{mid}_2 - \text{mid}_1$ 
3:  $n_3 \leftarrow \text{mid}_3 - \text{mid}_2$ 
4:  $n_4 \leftarrow \text{high} - \text{mid}_3$ 
5:  $\text{temp}_1[1 \dots n_1 + 1]$ 
6:  $\text{temp}_2[1 \dots n_2 + 1]$ 
7:  $\text{temp}_3[1 \dots n_3 + 1]$ 
8:  $\text{temp}_4[1 \dots n_4 + 1]$ 
9: for  $i \leftarrow 1$  to  $n_1$  do
10:    $\text{temp}_1[i] \leftarrow A[\text{low} + i - 1]$ 
11: end for
12: for  $j \leftarrow 1$  to  $n_2$  do
13:    $\text{temp}_2[j] \leftarrow A[\text{mid}_1 + j]$ 
14: end for
15: for  $k \leftarrow 1$  to  $n_3$  do
16:    $\text{temp}_3[k] \leftarrow A[\text{mid}_2 + k]$ 
17: end for
18: for  $l \leftarrow 1$  to  $n_4$  do
19:    $\text{temp}_4[l] \leftarrow A[\text{mid}_3 + l]$ 
20: end for
    ▷ The last element of each of the four sub-arrays is  $\infty$  so that we can easily find the minimum element among
    all sub-arrays
21:  $\text{temp}_1[n_1 + 1] \leftarrow \infty$ 
22:  $\text{temp}_2[n_2 + 1] \leftarrow \infty$ 
23:  $\text{temp}_3[n_3 + 1] \leftarrow \infty$ 
24:  $\text{temp}_4[n_4 + 1] \leftarrow \infty$ 
    ▷ Initialize the variables to point to the first element of each of the four sub-arrays
25:  $i \leftarrow 1$ 
26:  $j \leftarrow 1$ 
27:  $k \leftarrow 1$ 
28:  $l \leftarrow 1$ 
29:  $\text{start} \leftarrow \text{low}$ 
30: while ( $i \leq n_1 \parallel j \leq n_2 \parallel k \leq n_3 \parallel l \leq n_4$ ) do
31:   if ( $\text{temp}_1[i] \leq \text{temp}_2[j] \ \& \ \text{temp}_1[i] \leq \text{temp}_3[k] \ \& \ \text{temp}_1[i] \leq \text{temp}_4[l]$ ) then
    ▷ Element in  $\text{temp}_1[ ]$ 
    is minimum
32:      $A[\text{start}] \leftarrow \text{temp}_1[i]$ 
33:      $\text{start} \leftarrow \text{start} + 1$ 
34:      $i \leftarrow i + 1$ 
35:   else if ( $\text{temp}_2[j] \leq \text{temp}_1[i] \ \& \ \text{temp}_2[j] \leq \text{temp}_3[k] \ \& \ \text{temp}_2[j] \leq \text{temp}_4[l]$ ) then
    ▷ Element in  $\text{temp}_2[ ]$ 
    is minimum
36:      $A[\text{start}] \leftarrow \text{temp}_2[j]$ 
37:      $\text{start} \leftarrow \text{start} + 1$ 
38:      $j \leftarrow j + 1$ 
39:   else if ( $\text{temp}_3[k] \leq \text{temp}_1[i] \ \& \ \text{temp}_3[k] \leq \text{temp}_2[j] \ \& \ \text{temp}_3[k] \leq \text{temp}_4[l]$ ) then
    ▷ Element in  $\text{temp}_3[ ]$ 
    is minimum
40:      $A[\text{start}] \leftarrow \text{temp}_3[k]$ 
41:      $\text{start} \leftarrow \text{start} + 1$ 
42:      $k \leftarrow k + 1$ 
43:   else
    ▷ Element in  $\text{temp}_4[ ]$  is minimum
44:      $A[\text{start}] \leftarrow \text{temp}_4[l]$ 
45:      $\text{start} \leftarrow \text{start} + 1$ 
46:      $l \leftarrow l + 1$ 
47:   end if
48: end while
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