

Rod Cutting (Example)

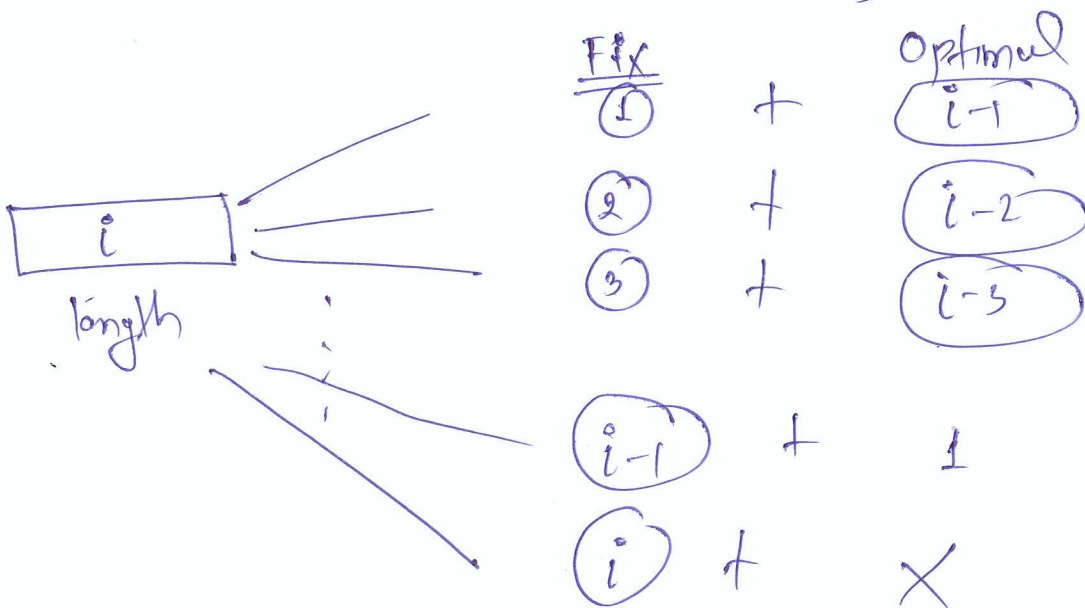
Length (i)	1	2	3	4	5	6	7	8
Price (p_i)	1	5	8	9	10	17	18 17	20

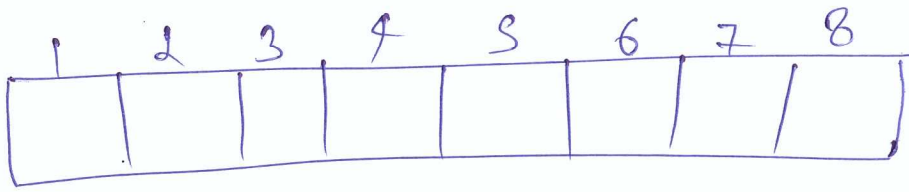
σ_i = Price of optimal cutting of rod till length i unit

~~$$\sigma_i = \text{Max} (p_1 + \sigma_{i-1}, p_2$$~~

$$\sigma_i = \text{Max}_{1 \leq k \leq i} \{ p_i + \sigma_{i-k} \}$$

$$= \text{Max} \{ p_1 + \sigma_{i-1}, p_2 + \sigma_{i-2}, p_3 + \sigma_{i-3}, \dots, p_i + \sigma_0 \}$$





Length = 8

- (1, 7)
- (2, 6)
- (3, 5)
- (4, 4)
- (5, 3)
- (6, 2)
- (7, 1)
- (8, 0)

One piece of length 5
+
Best price for length 3

Solution

$$(I) \quad \sigma_1 = 1$$

$$(II) \quad \sigma_2 = \text{Max} \left\{ \begin{array}{l} p_1 + \sigma_1 = 1 + 1 \\ p_2 = 5 \end{array} \right. = 5$$

$$(III) \quad \sigma_3 = \text{Max} \left\{ \begin{array}{l} p_1 + \sigma_2 = 1 + 5 = 6 \\ p_2 + \sigma_1 = 5 + 1 = 6 = 8 \\ p_3 + \quad = 8 + \quad = 8 \end{array} \right.$$

$$\text{IV. } \sigma_4 = \text{Max} \left\{ \begin{array}{l} p_1 + \sigma_3 = 1 + 8 = 9 \\ p_2 + \sigma_2 = 5 + 5 = 10 \\ p_3 + \sigma_1 = 8 + 1 = 9 \\ p_4 + \quad = 9 + \quad = 9 \end{array} \right. = 10$$

$$\text{V. } \sigma_5 = \text{Max} \left\{ \begin{array}{l} p_1 + \sigma_4 = 1 + 10 = 11 \\ p_2 + \sigma_3 = 5 + 8 = 13 \\ p_3 + \sigma_2 = 8 + 5 = 13 \\ p_4 + \sigma_1 = 9 + 1 = 10 \\ p_5 = 10 + \quad = 10 \end{array} \right. = 13$$

$$\text{VI. } \sigma_6 = \text{Max} \left\{ \begin{array}{l} p_1 + \sigma_5 = 1 + 13 = 14 \\ p_2 + \sigma_4 = 5 + 10 = 15 \\ p_3 + \sigma_3 = 8 + 8 = 16 \\ p_4 + \sigma_2 = 9 + 5 = 14 \\ p_5 + \sigma_1 = 10 + 1 = 11 \\ p_6 = 17 = 17 \end{array} \right. = 17$$

$$\text{VII. } \sigma_7 = \text{Max} \left\{ \begin{array}{l} p_1 + \sigma_6 = 1 + 17 = 18 \\ p_2 + \sigma_5 = 5 + 13 = 18 \\ p_3 + \sigma_4 = 8 + 10 = 18 \\ p_4 + \sigma_3 = 9 + 8 = 17 \\ p_5 + \sigma_2 = 10 + 5 = 15 \\ p_6 + \sigma_1 = 17 + 1 = 18 \\ p_7 + \quad = 17 = 17 \end{array} \right. = 18$$

VIII

$\sigma_8 = \text{Max}$

$$p_1 + \sigma_7 = 1 + 18 = 19$$

$$p_2 + \sigma_6 = 5 + 17 = \textcircled{22}$$

$$p_3 + \sigma_5 = 8 + 13 = 21$$

$$p_4 + \sigma_4 = 9 + 10 = 19$$

$$p_5 + \sigma_3 = 10 + 8 = 18$$

$$p_6 + \sigma_2 = 17 + 5 = \textcircled{22}$$

$$p_7 + \sigma_1 = 17 + 1 = 18$$

$$p_8 = 20 = 20$$

22

$p_2 + \sigma_6$

2

5

6

17

$p_6 + \sigma_2$

6

17

2

5

Find the pieces using

Travasy Bank